Archiwum Motoryzacji vol 69 PIMOT

INNOVATIVE METHOD OF EXPERIMENTAL IDENTIFICATION OF THE DEFORMATION OF VEHICLE TYRES

PART 2: NOVEL TRANSDUCER FOR MEASURING THREE VEHICLE TYRE DEFORMATION COMPONENTS

PIOTR DUDZIŃSKI[1] , JAKUB CHOŁODOWSKI[2]

Wrocław University of Technology

Summary

The state of the art in the field of transducers and tyre deformation measurement methods tells of the high importance of this issue. The "tyre deformation" may be analysed from the point of view of both the global deformation of the tyre structure and the local deformation of single tread lugs. So far, there is no all-purpose transducer that would be capable of identifying simultaneously all the tyre deformation components that would be interesting for researchers in all the above aspects. If it is considered sufficient to define the tyre deformation by the value of displacement of a selected tyre point in relation to the wheel rim, then the deformation can be measured with the use of a device built in accordance with the original concept presented in this article, i.e. a mechanical contacting transducer. The transducer is intended to measure the tyre deformation by separately determining the three principal (tangential, lateral, and radial in relation to the wheel disc) deformation components, which are to be recorded at the same time. The article presents a general mathematical model of the device, with highlighting the diversity of methods of describing the model in a formal way and illustrating them by possible examples. Illustrative different paths of implementing the concept of the device have been shown as well. A measuring device built in accordance with the idea may constitute a tool for identifying the mechanical tyre characteristics, which might be then used inter alia as input data at simulation research on vehicles, e.g. for modelling the vehicle body vibrations or for analysing the energy absorption at vehicle motion.

Keywords: tire deformation, measurement of tire deformation components, radial/lateral/tangential component of tire deformation, multiple-sensor 3D tire deformation transducer

[1] Wrocław University of Technology, Faculty of Mechanical Engineering, Chair of Construction Machinery and Industrial Vehicles Engineering, bud. B-5 ul. Łukasiewicza 7/9, 50-370 Wrocław, e-mail: piotr.dudzinski@pwr.edu.pl, tel. +48 71 321 53 96

[2] Wrocław University of Technology, Faculty of Mechanical Engineering, Chair of Construction Machinery and Industrial Vehicles Engineering, bud. B-5 ul. Łukasiewicza 7/9, 50-370 Wrocław, e-mail: jakub.cholodowski@pwr.edu.pl, tel. +48 500 22 11 96

1. Introduction

In a review of the state of the art discussed in part 1 of this article, conspicuous is the unavailability of a tyre deformation transducer that would be free of drawbacks of any kind. In particular, there is no device that would make it possible to carry out comprehensive measurements of tyre deformations where a relatively low level of complication of the computation algorithm would be simultaneously maintained [6]. As an attempt to find a compromise between these two features, an original concept of a mechanical contacting transducer has been proposed here. The state of the art in the field of transducers for tyre deformation measurements illustrates the great extent of this issue [6]. In consideration of the ambiguity of the problem, the process of designing a tyre deformation transducer was started from defining the design basis, which would determine the shape of the concept that was to be developed that has been described in this article.

2. Design basis

In this paper, tyre deformation is defined as the displacement of a selected point on the inner cylindrical tyre surface in relation to a reference frame attached to the wheel rim (x, y, z, see Fig. 1). The transducer presented herein is chiefly to be used for tyre deformation measurements in the conditions of rig testing of vehicle wheels either driven or freely rolling on road surfaces of various kinds.

In the configuration as presented herein (Fig. 2), the test rig makes it possible to test motor vehicle and industrial vehicle tyres with outer diameters of not more than 680 mm. Basically, the test road surface is represented by the surface of movable metal plate (3) used to measure the "tyre-road" reaction components. Moreover, the identification may be carried out for the process of tyre interaction with a ground surface of another type, flexible or rigid, e.g. soil placed in a pan-like container rigidly fixed to plate (3) or a plate having the structure of asphalt and attached to the rig metal plate likewise. The radial deflection of the tyre under test is forced by hydraulic cylinder (6) or screw (7), used to adjust the position of rig frame (2) relative to plate (3). The wheel with the tyre under test (1) may either be driven by hydraulic motor (4) incorporated in the wheel axle unit or freely roll on plate (3) when the latter is moved by hydraulic cylinder (5). The length of plate (3) is such that the tyre can be tested under quasi-static load conditions at a wheel driving or rolling velocity not exceeding 1.2 m/s. The tyre operation at different sideslip angles is enabled by cylinder (8) and rotary module (9). When the tyre is positioned in the plane of motion of plate (3), it operates at a zero sideslip angle and is only deformed in the tangent and radial direction. At non-zero sideslip angle values, it is additionally subjected to lateral deformation.

During the tyre deformation measurement on the test rig as shown above, each of the three principal deformation components, i.e. the tangential, lateral, and radial component (δx, δy, and δz, respectively, see Fig. 1), should be recorded simultaneously. The planned measuring range of the transformer (Table 1) has been estimated on the grounds of literature data [2, 12, 19] and manufacturer's tyre specifications.

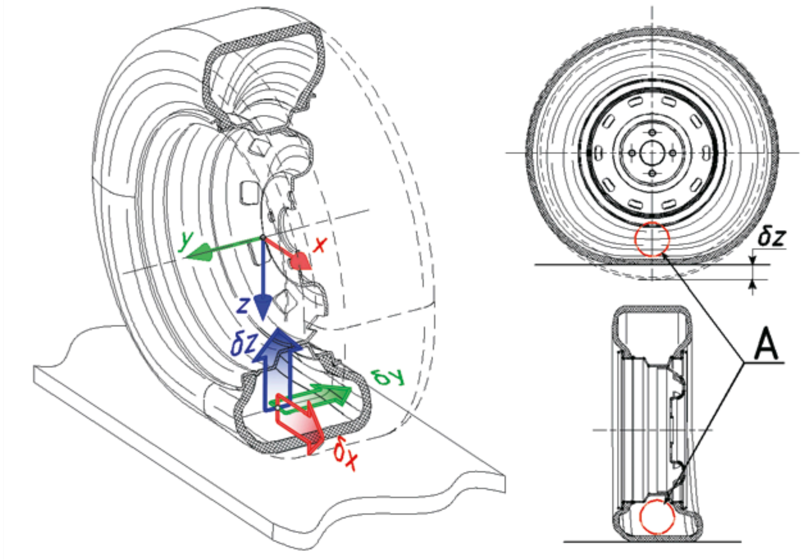

Fig. 1. Illustration of the design basis against the background of a road wheel in the tyre deformation conditions: x, y, z – coordinate system attached to the rim as a reference frame for the tyre deformation measurement; δx, δy, δz – tangential, lateral and radial component of the tyre deformation, respectively; A – location of the multiple-sensor 3D tyre deformation transducer assembly

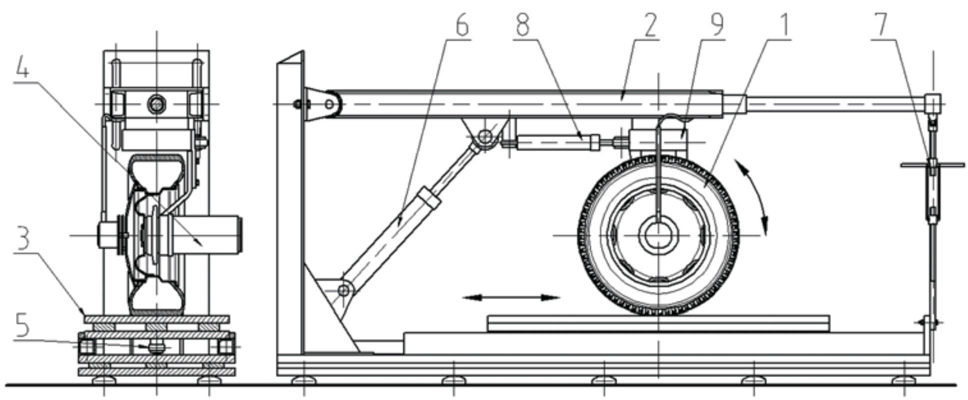

Fig. 2. Test rig for the investigation of static and dynamic properties of tyres: 1 – wheel with the tyre under investigation; 2 – frame; 3 – plate simulating the test road surface, enabling the measurements of reaction components; 4 – hydraulic motor propelling the wheel (1); 5 – hydraulic cylinder propelling the plate (3); 6, 7 – hydraulic cylinder and turnbuckle screw for vertical loading and unloading the wheel (1); 8, 9 – hydraulic cylinder and module for steering the wheel (1)

Table 1. Anticipated range of values of individual tyre deformation components (see Fig. 1), estimated for passenger cars and light trucks on the grounds of literature data [2, 12, 21] and manufacturer's tyre specifications

Deformation component	Min. value	Max. value	Amplitude (peak-to-peak)
	[mm]		
Tangential (δx)	−8.0	+8.0	16.0
Lateral (δy)	−36.0	+36.0	72.0
Radial (δz)	−31.0	+5.5	36.5

For the harmful external impacts on the measurement results to be minimized, e.g. for the errors that might be caused by the flexibility of test rig components to be avoided, and for tyre deformation measurements to be possible even for the tyre interacting with deformable ground, the transducer dimensions and construction should enable the transducer to be installed inside the tyre under test (within zone A, Fig. 1). The installation would be more convenient if the transducer were designed in the form of a compact package of sensors, whose installation would consist of a limited number of operations. The coordination of measurement results in the domain of angular wheel positions would make it possible to present the results in the form of deformation maps, thanks to which their correctness might be assessed in an easier way both by comparing them with the results presented in a similar form in the literature [12, 21] and by examining them with reference to the wheel operation conditions simulated on the test rig. It would be desirable, therefore, if components of the package of sensors could be used at least for the preliminary evaluation of the angular position of the wheel.

3. Solution version adopted; the theoretical model

At the beginning of the designing process, a group of competing transducer ideas was compiled, based on the weak and strong points of the solutions known to date. Then, the most advantageous idea was selected by appraising each of them based on various criteria, e.g. compactness, anticipated metrological parameters, or possibility of carrying out remote measurements. In consideration of the difficulties encountered in the simultaneous meeting of all the criteria, the concept of a mechanical contacting transducer was chosen as the best compromise, where the following three areas, interconnected with each other, could be discerned:

- mechanical structure;
- package of sensors;
- mathematical model with an algorithm of processing the measuring signals.

The transducer's mechanical structure is based on the main linkage, which connects the inner surface of the tyre bottom with the wheel rim. For the sake of compact transducer design and for the impact of the resistance to motion in the kinematic pairs on the tyre deformation pattern to be minimized, the option with the minimum number of members and kinematic pairs (Fig. 3) was chosen as the most advantageous solution in the mechanism synthesis process. The movements of the tyre surface in relation to the wheel rim are mapped by a measuring tip (3, Fig. 3), which remains in contact with a selected point on the tyre surface ($K_p \rightarrow K_p$', Fig. 3). On the other hand, the transducer base assembly (1, Fig. 3) is permanently fixed to the rim; in consequence, the tyre movements are represented by changes in the mechanism configuration. The radial deflection (δz) of the tyre results in a translational motion in the sliding pair connecting the transducer probe (3, Fig. 3) with the base sleeve (2, Fig. 3). The presence of the other deformation components ($\delta x, \delta y$) results in additional tilting of the probe together with the sleeve. The values of the displacements in the kinematic pairs unequivocally define the position of the measuring tip in the coordinate system attached to the rim of the wheel under test. Individual components of the package of sensors are a source of signals representing the displacements in individual kinematic pairs. The acquisition of these signals leads to the obtaining of input data for the algorithm of computing individual tyre deformation components.

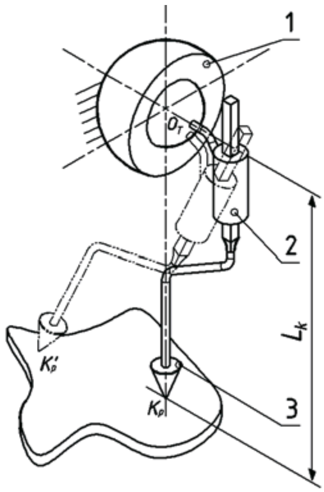

Fig. 3. Kinematic diagram of the main linkage of the multiple-sensor 3D tyre deformation transducer: 1 – transducer base attached to the rim; 2 – linking element (base sleeve) enabling the transducer probe to tilt and to move along its longitudinal axis; 3 – transducer probe with a measuring tip attached to the inner tyre surface

The algorithm is based on a mathematical model, where the main linkage (Fig. 3) is represented by vector $\boldsymbol{v}$ (Fig. 4), whose origin is situated at the centre of the swivel joint between the base sleeve and transducer base (O_T, Fig. 3) and whose endpoint is associated with the measuring tip (Kp, Fig. 3). The presence of an articulated (swivel) joint between the base sleeve and the transducer base results in the fact that vector $\boldsymbol{v}$ can freely rotate in relation to the x_P, y_P, z_P coordinate system attached to the transducer base. The mobility of the measuring tip in relation to the base sleeve translates into variability of length of vector v. Thanks to the introduction of an additional coordinate system x_T, y_T, z_T rotating with vector v, i.e. attached to the base sleeve, the problem of defining the coordinates of the measuring tip in relation to the transducer base was divided into two stages.

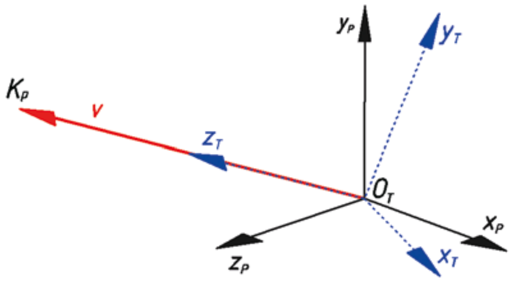

Fig. 4. Mathematical model of the main linkage of the multiple-sensor 3D tyre deformation transducer: v – rotating vector representing the probe, having variable length and orientation due to movement of the probe tip relative to the rim; x_T, y_T, z_T – coordinate system with its z_T axis coinciding with the longitudinal axis of the probe; x_P, y_P, z_P – coordinate system attached to the transducer base and the wheel rim

The first stage was dedicated to defining the coordinates of vector T_v (1), i.e. vector $\boldsymbol{v}$ in the coordinate system denoted by index T, which just meant determining the distance between the measuring tip and the centre of rotation of the base sleeve in relation to the transducer base assembly (L_k, Fig. 3).

$$^{T}\boldsymbol{v} = [0 \quad 0 \quad L_k] \tag{1}$$

The second stage was aimed at determining the mutual angular orientation of the base sleeve and the transducer base, i.e. the coordinate systems denoted by indices T and P, respectively. In the description of this orientation, the use of one of the two notations discussed in a subsequent part of this article was proposed; in consequence, two alternative descriptions of the vectorial model of the transducer can be used.

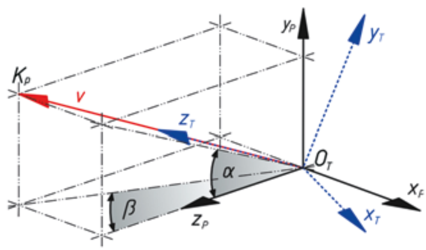

Fig. 5. Representation of the orientation of vector v relative to the stationary coordinate system x_P, y_P, z_P (attached to the transducer base), with the use of angles α and β between the projections of vector v on planes y_P, z_P and x_P, z_P, respectively, and axis z_P

In the first notation (Fig. 5), the orientation of the base sleeve in relation to the transducer base assembly is described by two angles α and β, defined by axis z_P and projections of vector $\boldsymbol{v}$ on planes y_P, z_P and x_P, z_P. From the linkage geometry point of view, these angles are the angles of rotation of the base sleeve relative to axes x_P and y_P. The coordinates of the measuring tip ${}^{P}\boldsymbol{v}$ (2) in the coordinate system attached to the transducer base assembly are obtained by transformation of vector ${}^{T}\boldsymbol{v}$ (1) to the coordinate system denoted by index P, according to equations (3, 4, and 5).

$$ {}^{P}\boldsymbol{v} = [v_x \quad v_y \quad v_z] \tag{2}$$

$$ v_x = L_k \operatorname{tg} \alpha \sqrt{\frac{1}{1+\operatorname{tg}^2\alpha+\operatorname{tg}^2\beta}} \tag{3}$$

$$ v_y = L_k \operatorname{tg} \beta \sqrt{\frac{1}{1+\operatorname{tg}^2\alpha+\operatorname{tg}^2\beta}} \tag{4}$$

$$ v_z = L_k \sqrt{\frac{1}{1+\operatorname{tg}^2\alpha+\operatorname{tg}^2\beta}} \tag{5}$$

In the second notation, a description formulated with the use of unit quaternions has been adopted. The quaternion is defined as a complex number $\boldsymbol{q}$ belonging to a four-dimensional space and unequivocally described by four parameters (6):

$$ \boldsymbol{q} = [q_1 \quad q_2 \quad q_3 \quad q_4] \tag{6}$$

The parameter q_1 is the real part of the quaternion while the other parameters constitute its imaginative part. Similarly to the case of a set of two-parameter complex numbers, the notion of a conjugate quaternion $\bar{\boldsymbol{q}}$ is defined in accordance with equation (7):

$$ \bar{\boldsymbol{q}} = [q_1 \quad -q_2 \quad -q_3 \quad -q_4] \tag{7}$$

A unit quaternion is the quaternion whose norm, determined from equation (8), is equal to 1:

$$ \| \boldsymbol{q} \| = \sqrt{q_1^2 + q_2^2 + q_3^2 + q_4^2} \tag{8}$$

The theoretical foundations of the notion of quaternions as well as definitions of the mathematical operations carried out in this space have been more comprehensively described e.g. in publications [3, 7, 10, 14, 15].

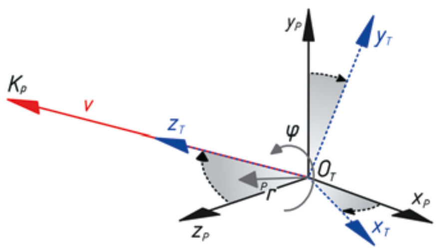

Fig. 6. Representation of the orientation of vector v relative to the xP, yP, zP coordinate system, using unit quaternions: φ – value of the angle of rotation of the P reference frame around the unit vector ${}^{P}r$, needed for the corresponding axes of the P and T reference frames to coincide, after [15]

The use of unit quaternions to represent the orientation of two coordinate systems in relation to each other has been illustrated by an example shown in Fig. 6. If the unit quaternion ${}^{P}_{T}\boldsymbol{q}$ describes the orientation of the coordinate system denoted by index T in relation to the P coordinate system, then each of the four parameters of the quaternion is a function of angle φ and coordinates of the unit vector ${}^{P}\boldsymbol{r}$ (9), around which the P coordinate system has to be rotated by angle φ in a single operation for axes x_P, y_P, and z_P to coincide with axes x_T, y_T, and z_T, respectively (10). The vector ${}^{P}\boldsymbol{r}$ is defined by its coordinates in the P coordinate system:

$$ {}^{P}\boldsymbol{r} = [r_x \quad r_y \quad r_z] \tag{9} $$

$$ {}^{P}_{T}\boldsymbol{q} = \left[\cos\frac{\varphi}{2} \quad {}^{P}\boldsymbol{r}\sin\frac{\varphi}{2}\right] = \left[\cos\frac{\varphi}{2} \quad r_x\sin\frac{\varphi}{2} \quad r_y\sin\frac{\varphi}{2} \quad r_z\sin\frac{\varphi}{2}\right] \tag{10} $$

The inverse relation, i.e. the orientation of the P coordinate system relative to the T system, is described by quaternion ${}^{T}_{P}\boldsymbol{q}$, which is a quaternion conjugate with ${}^{P}_{T}\boldsymbol{q}$ (11):

$$ {}^{T}_{P}\boldsymbol{q} = {}^{P}_{T}\overline{\boldsymbol{q}} \tag{11} $$

Equation (11) as against (7) may be interpreted as follows: If corresponding axes of the P and T coordinate systems become coincident in result of a rotation of the P system around vector ${}^{P}\boldsymbol{r}$ by angle φ, then an identical effect will take place when the T system rotates in relation to vector ${}^{P}\boldsymbol{r}$ in the opposite direction, i.e. by angle $-\varphi$. The sine function is an odd one; hence, the imaginative parameters of quaternion ${}^{T}_{P}\boldsymbol{q}$ will have values opposite to those of the corresponding parameters of ${}^{P}_{T}\boldsymbol{q}$.

If the notation of unit quaternions is used, the transformation of vector ${}^{T}\boldsymbol{v}$ (1) to the coordinate system attached to the transducer base may be done directly in the quaternion space by carrying out two multiplication operations (12). The operation of multiplication in the quaternion space may be illustrated by an example presented as equation (13) and it is a prerequisite for it that the three-dimensional vectors ${}^{T}\boldsymbol{v}$ and ${}^{P}\boldsymbol{v}$ must be transferred to the quaternion space (14, 15).

$$ {}^{P}\boldsymbol{q}_{\boldsymbol{v}} = {}^{T}_{P}\boldsymbol{q} \otimes {}^{T}\boldsymbol{q}_{\boldsymbol{v}} \otimes {}^{T}_{P}\overline{\boldsymbol{q}} \tag{12} $$

$$ \boldsymbol{c} = \boldsymbol{a} \otimes \boldsymbol{b} = [a_1 \quad a_2 \quad a_3 \quad a_4] \otimes [b_1 \quad b_2 \quad b_3 \quad b_4] = \begin{bmatrix} a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 \\ a_1b_2 + a_2b_1 + a_3b_4 - a_4b_3 \\ a_1b_3 - a_2b_4 + a_3b_1 + a_4b_2 \\ a_1b_4 + a_2b_2 - a_3b_3 + a_4b_1 \end{bmatrix}^T \tag{13} $$

$$ {}^{T}\boldsymbol{q}_{\boldsymbol{v}} = [0 \quad {}^{T}\boldsymbol{v}] = [0 \quad 0 \quad 0 \quad L_k] \tag{14} $$

$$ {}^{P}\boldsymbol{q}_{\boldsymbol{v}} = [0 \quad {}^{P}\boldsymbol{v}] = [0 \quad v_x \quad v_y \quad v_z] \tag{15} $$

4. Concepts of implementing the idea of a mechanical contacting transducer of tyre deformation

Based on the theoretical model, an attempt to implement the idea of a mechanical contacting transducer for tyre deformation measurements was made by proposing the construction of two generations of such a transducer. In both design solutions, the transducer (Figs. 7a and 10) as a whole was introduced into the tyre interior through an airtight lock prepared in the wheel rim (Figs. 7b, 11, and 12). The transducer base assembly, being the only transducer component directly connected with the wheel rim, was mounted in the lock chamber, thanks to which the number of transducer installation operations was considerably reduced. In the designs of both generations, the mechanical structure provided a support for different transducer sets. For each of the transducer generations, the main linkage was designed in a different form and a different version of the mathematical description was used.

First-generation transducer

The first-generation transducer (Fig. 7) is an implementation of the mathematical model illustrated in Fig. 5.

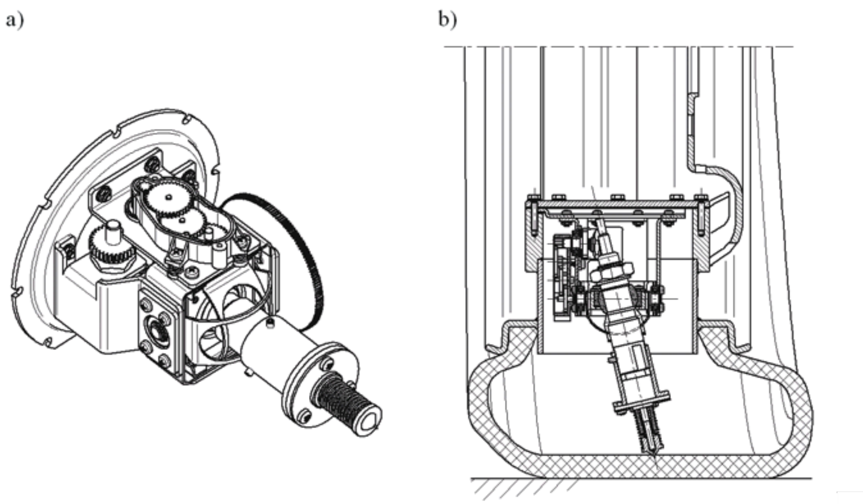

Fig. 7. Multiple-sensor 3D tyre deformation transducer of the first generation: a) isometric view of the transducer; b) transducer mounted inside a wheel whose tyre is deformed under complex loading conditions

The L_k parameter value is represented by the signal produced by a potentiometric linear position sensor with a sliding probe. For the transducer dimensions to be reduced to a minimum, this sensor has been incorporated in the main linkage. Thus, it plays the role of the only kinematic pair that provides mobility of the measuring tip along the transducer probe axis. To prevent any displacement of the measuring tip on the tyre surface during measurements, the tip has been formed as a cone with a sharp end, pressed by a spring against the tyre surface (Fig. 7) or, in another version, a small-diameter ball mating with a hemispherical socket fixed at a selected point to the tyre bottom [7]. The potentiometer housing is supported by a swivel bearing retained in the transducer base assembly; it is fixed to the inner bearing ring through the base sleeve and a mounting head, thanks to which the potentiometer housing position relative to the bearing rotation centre can be adjusted and, in consequence, the transducer can be adapted to testing tyres with different profile heights. When the mounting head is brought to a position as required, it is fixed in relation to the base sleeve by means of set screws.

The angles α and β are measured by a pair of potentiometric angular position sensors together with components of the mechanisms converting the tilt of the base sleeve (1, Fig. 8) into rotation of the shafts of the potentiometric sensors (2, Fig. 8). A key role in the torque transmission from the base sleeve to the shaft of the angular position transducer is played by the arm pivoting on supports provided in the transducer base assembly. The axis of rotation of the arm goes through the centre of rotation of the inner ring of the swivel bearing that supports the base sleeve. The movements of the base sleeve and the pivoting arm are coupled with each other by a yoke-slider kinematic pair, thanks to which the projection of the base sleeve motion on the plane normal to the arm rotation axis can be mapped by the arm. The arm rotation is transmitted to the shaft of the angular position transducer via a multi-stage multiplying gear train.

The algorithm of the fusion of the measuring data described by equations (3, 4, 5) and of the calculation of individual tyre deformation components is to be implemented in

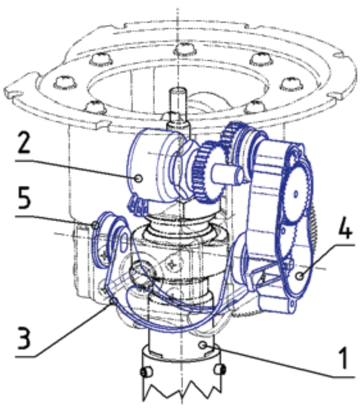

Fig. 8. Multiple-sensor 3D tyre deformation transducer of the first generation. The mechanism for converting transducer probe's tilt into rotation of the shaft of the potentiometric angular position sensor: 1 – base sleeve, in which the housing of the potentiometric sensor measuring the linear position of the transducer probe is fixed; 2 – potentiometric sensor to measure the angular position of the base sleeve relative to the axis tangent to the wheel circumference (x_p); 3 – pivoting arm; 4 – multi-stage gear train; 5 – ball bearing to support the pivoting arm

a computer, following the acquisition of signal samples, carried out with the use of a laboratory measuring amplifier (Fig. 12).

Second-generation transducer

The second-generation transducer (Fig. 9) is an implementation of the mathematical model explained in Fig. 6. The modular transducer design will facilitate the replacement of components of the package of sensors in case that any of them fails or that sensor elements with better metrological characteristics appear on the market.

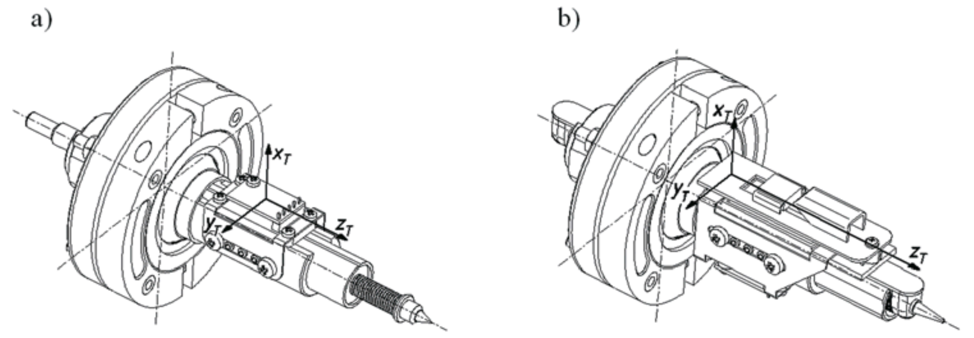

Fig.9. Multiple-sensor 3D tyre deformation transducer of the second generation: a) option with a potentiometric displacement sensor; b) option with an optic displacement encoder

The shape and dimensions of the transducer base, swivel bearing, and base sleeve assembly have been designed with keeping in mind a requirement that the use of various sensors for determining the position of the measuring tip relative to the base sleeve should be possible. The adoption of multi-optional solutions was planned as early as at the designing stage. An alternative for the potentiometric position sensor (Fig. 9a) may be an inductive linear variable displacement transducer (LVDT) or a unique module with an optic displacement encoder (Fig. 9b), designed by the authors.

The design of the base assembly of the second-generation transducer has been modified so that the base consists of two separate parts. Thanks to this, a possibility to choose the measurement datum has been obtained (Fig. 10). At the beginning of the operation of installing the transducer inside the tyre, the datum-setting part of the transducer base is positioned as required relative to the airtight lock and fixed by the friction force between the lock walls and expanding fasteners provided between the datum-setting base part and the lock walls. Then, the swivel bearing retainer, together with all the other transducer components, including the base sleeve seated in the bearing as well as the package of sensors, is fastened to the datum-setting base part.

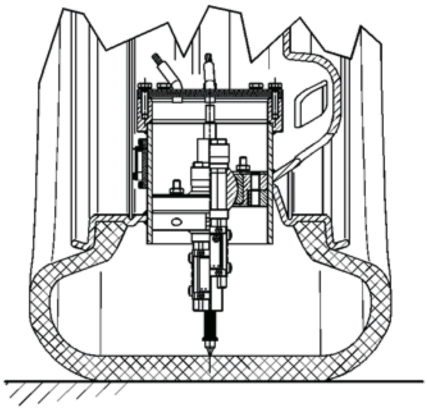

Fig. 10. Multiple-sensor 3D tyre deformation transducer of the second generation: adjustable positioning of the transducer probe centre of rotation by changing the position of the datum-setting part of the transducer base

The parameters of the unit quaternion that represents the mutual orientation of the base sleeve and the transducer base assembly are estimated by a non-contact method with the use of a pair of synchronously operating AHRS (Attitude and Heading Reference System) modules. The existing applications of such modules include e.g.:

- aviation and underwater technology, in trajectory stabilization and vehicle levelling systems in unmanned aerial and underwater vehicles ([1] and [13], respectively);
- behavioural biometry of living organisms, including humans [19];
- analysis of motion of industrial robots, carried out to evaluate the usability of inertial sensors in the construction of feedback paths in the robot control systems [18].

A typical AHRS module consists of a module of IMU (Inertial Measurement Unit) sensors and a computing unit. In a special case, the IMU module may be made as a triad of orthogonal accelerometers, gyroscopes and magnetometers. The values representing the sensor signal levels are fed as input data to the computing unit, which estimates, on these grounds and in accordance with a pre-programmed algorithm, the values of the parameters describing the current orientation of the coordinate system attached to the sensors' sensitivity axes relative to the system attached to the terrestrial gravitational and magnetic field vectors. The algorithms developed for the purposes of estimation of the orientation-representing parameters in the quaternion notation are covered *inter alia* in publications [1, 11, 15, 16, 17, 20].

In the realities of the second-generation tyre deformation transducer, the IMU modules have been arranged as shown in Fig. 11 and each of them has been composed of sensors made to the MEMS (Micro-Electro-Mechanical Systems) technology. The IMU sensors attached to the transducer base assembly are a source of information about the current orientation of the wheel rim (x_P, y_P, z_P) relative to the reference frame attached to the vectors of the terrestrial gravitational and magnetic field (x_Z, y_Z, z_Z), where the orientation is described by quaternion ${}^{Z}_{P}\boldsymbol{q}$. Quaternion ${}^{T}_{Z}\boldsymbol{q}$, on the other hand, which describes the orientation of the base sleeve (x_T, y_T, z_T) in the same reference frame, is estimated on the grounds of the signals produced by the sensors of the module installed on the base sleeve surface. Both the IMU modules are interconnected with each other through a common computing unit, i.e. a microcontroller in this case. The microcontroller cyclically samples the signals received from all the sensors and thus obtains the data that make it possible to estimate the parameters of quaternion ${}^{T}_{P}\boldsymbol{q}$ (11), which describes the mutual orientation between the transducer base and the base sleeve. The said parameters are estimated by calculating the product (16):

$$ {}^{T}_{P}\boldsymbol{q} = {}^{Z}_{P}\boldsymbol{q} \otimes {}^{T}_{Z}\boldsymbol{q} \tag{16} $$

The cyclically estimated parameters of the unit quaternion ${}^{T}_{Z}\boldsymbol{q}$ may also be used for rough determining of the angular position of the wheel during the experiment.

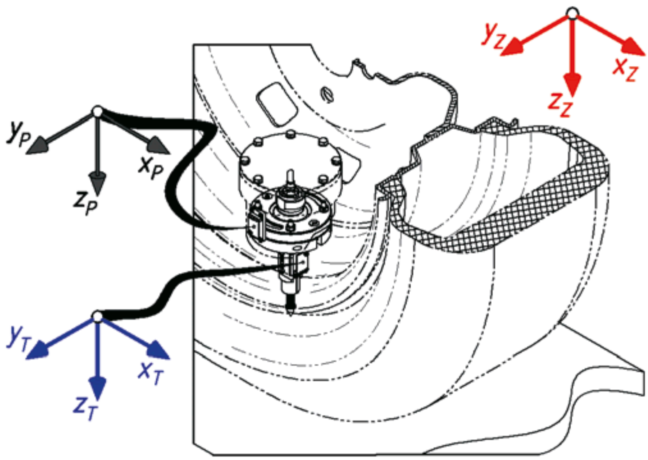

Fig. 11. Multiple-sensor 3D tyre deformation transducer of the second generation: arrangement of the IMU modules used for determining the parameters of the unit quaternion that describes the mutual orientation between the base sleeve and the transducer base (i.e. the wheel rim)

Fig. 12 shows a block diagram where the algorithms of operation of tyre deformation transducers of both generations are confronted with each other. As regards the second-generation transducer, the algorithm presented shows the operation of a transducer where the computing unit not only coordinates the operation of the IMU modules but also cyclically samples the signal received from the linear position sensor. This means that a complete set of data required for the calculation of the position and displacement of the measuring tip in the reference frame attached to the wheel rim is available to the computing unit, thanks to which the necessity of processing the measuring data at the successive data processing stages has been eliminated (compare 3 I – 4 1 with 3 II – 4 II, Fig. 12).

5. Recapitulation

The article presents concepts of two generations of tyre deformation transducers. The novelty of the solutions discussed here as against the state of the art in this field at the instant when the design work was undertaken provided grounds for applying for patent protection for them [7, 8].

The transducer concept developed is in conformity with the design basis having been discussed in the early part of this paper, thanks to which it is suitable for laboratory rig testing of vehicle tyres. The first of the solutions proposed is characterized by very clear algorithm of the fusion of measurement data; the other one, although more complicated from the algorithm point of view, is conspicuous for its simplified mechanical design. The modular structure of both of them offers long-term prospects of their development.

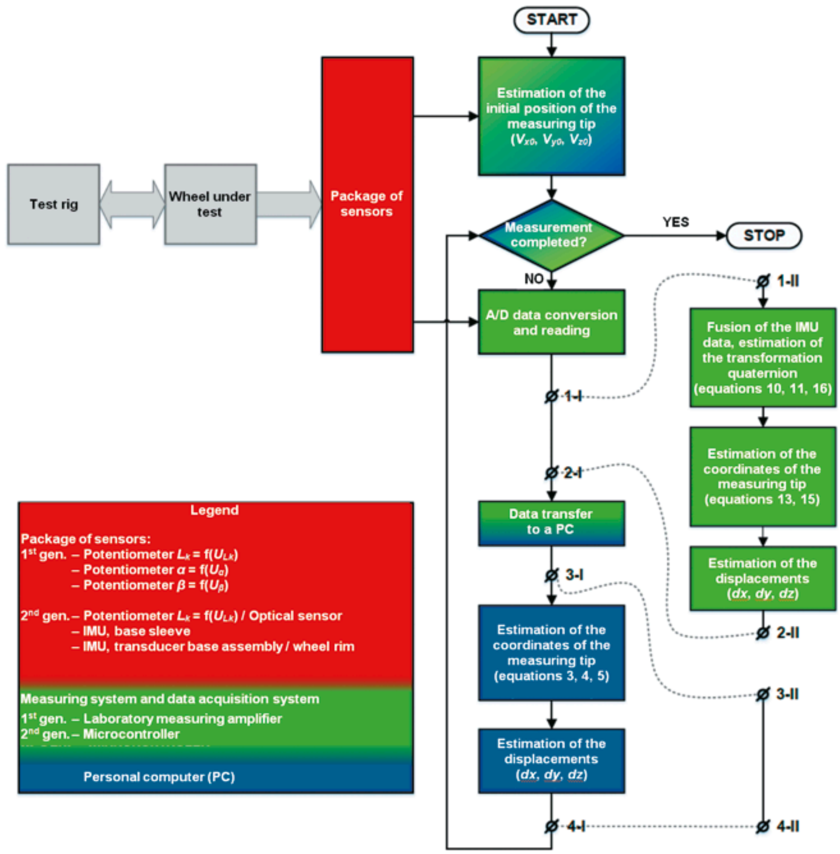

Fig. 12. Generalized block diagram of the algorithm of operation of a multiple-sensor 3D tyre deformation transducer, showing the differences between the algorithms implemented by the transducers of the first and second generation:
X-I – algorithm sections exclusively implemented by the 1st generation transducer;
X-II – algorithm sections exclusively implemented by the 2nd generation transducer

The transducer may be improved by replacing individual components of the package of sensors with new sensors having better metrological characteristics as they appear on the market and/or by developing the software used for the fusion of the measuring data, e.g. by testing various algorithms of estimation of the parameters defining the orientation of the base sleeve and transducer base, changing the integration methods employed in the algorithms, or striving for reductions in the list of instructions in the transducer operation management program, which would result in maximizing the iteration frequency [20]. Thanks to the modular transducer design, it may be easily adapted, by minor modifications, for the measurements of deformations of other structures similar to vehicle wheels.

Due to considerable overall dimensions and mass of the device presented, its adaptation for use in typical conditions of motor vehicle operation seems to be difficult. The dynamic balancing of a road wheel would require the use of a balance weight with a mass significantly exceeding that of typical tyre weights, concentrated on the rim side opposite to the transducer fastening place. Therefore, if the method proposed were to be used for *in situ* tests, then the tests should be carried out at low vehicle velocities, e.g. for low-speed industrial vehicles, for which dynamic wheel balancing is not required. In the widest time horizon, however, the transducer built in accordance with the concept presented may be a starting point for the construction of a similar device of even smaller geometrical dimensions and mass, preferably intended for non-contact measurements, i.e. a device whose features would make it usable for not too invasive installation in commercially available vehicles for the purposes of ongoing monitoring of the technical condition of the tyre or the current state of the road surface or as equipment supporting the operation of on-board vehicle safety systems such as ABS or traction control systems.

References

[1] Adiprawita W., Ahmad A. S. and Sembiring J.: *Development of AHRS* (*Attitude and Heading Reference System*) *for Autonomous UAV* (*Unmanned Aerial Vehicle*). International Conference on Electrical Engineering and Informatics, Bandung, Indonesia, 17–19 June 2007, Institute of Electrical and Electronics Engineers, 2007, pp. 714–717.

[2] Andrzejewski R.: *Dynamika pneumatycznego koła jezdnego* (*Dynamics of a wheel and pneumatic tyre assembly*). Warszawa: WNT, 2010.

[3] Bekir E.: *Introduction to Modern Navigation Systems.* World Scientific Publishing Co. Pte. Ltd., Singapore 2007.

[4] Chołodowski J.: *Opracowanie metody pomiaru deformacji opony w procesie jej interakcji z dowolnym podłożem* (*Development of a method of measuring the tyre deformation in the process of interaction of the tyre with ground of any kind*). Engineer's graduation work, supervised by Prof. Piotr Dudziński, Ph. D. Eng., Wrocław University of Technology, Wrocław 2013.

[5] Chołodowski J.: *Projekt przetwornika nowej generacji do pomiaru deformacji opony* (*Design of a transducer of new generation for tyre deformation measurements*). Master's graduation work, supervised by Prof. Piotr Dudziński, Ph. D. Eng., Wrocław University of Technology, Wrocław 2014.

[6] Chołodowski J., Dudziński P.: *Innowacyjna metoda identyfikacji eksperymentalnej deformacji opon pojazdów. Część 1: Systematyka metod pomiaru deformacji opon* (*Innovative method of experimental identification of the deformation of vehicle tyres. Part 1: Systematics of the known tyre deformation measurement methods*). The Archives of Automotive Engineering – Archiwum Motoryzacji, vol. 67, 2/2015.

[7] Diebel J.: *Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors*. Stanford University, Stanford 2006.

[8] Dudziński P., Chołodowski J.: *Czujnik do pomiaru przemieszczeń, zwłaszcza elementów ogumienia względem obręczy koła jezdnego* (*Sensor for the measurement of displacements, typically displacements of tyre elements in relation to the wheel rim*). Patent application: P.409800, Warszawa 2014.

[9] Dudziński P., Chołodowski J.: *Czujnik do pomiaru przemieszczeń, zwłaszcza elementów ogumienia względem obręczy koła jezdnego* (*Sensor for the measurement of displacements, typically displacements of tyre elements in relation to the wheel rim*). Patent application: P.409801, Warszawa 2014.

[10] Farrell J. A.: *Aided navigation, GPS with High Rate Sensors*. The McGraw-Hill Companies, USA 2008, Annex D, pp. 501–513.

[11] Fresk E.: *Quaternion based Extended Kalman Filter as an Attitude and Heading Reference System*. Lulea University of Technology, Lulea, Sweden 2012.

[12] Green R. W.: *A Non-contact Method for Sensing Tire Contact Patch Deformation Using a Monocular Vision System and Speckled Image Tracking*. Master's graduation work, Auburn University, Auburn, USA 2011.

[13] Kim H. S. Choi H. S., Yoon J. S. and Ro P. I.: *Study on AHRS Sensor for Unmanned Underwater Vehicle*. International Journal of Ocean System Engineering, Korean Society of Ocean Engineers, 3, 1, 2011, pp. 165–170.

[14] Kuipers J. B.: *Quaternions and rotation sequences*. Geometry, Integrability and Quantization, Varna, Bulgaria, 1–10 Sept. 1999, Coral Press, Sofia 2000, pp. 127–143.

[15] Madgwick S. O. H.: *An efficient orientation filter for inertial and inertial/magnetic sensor arrays*. 30 April 2010, on-line: https://www.samba.org/tridge/UAV/madgwick_ internal_report.pdf, last visited on 23 March 2014.

[16] Madgwick S. O. H., Harrison A. J. L. and Vaidyanathan R.: *Estimation of IMU and MARG orientation using a gradient descent algorithm*. 2011 IEEE International Conference on Rehabilitation Robotics Rehab Week Zurich, Zurich, Switzerland, 29 June–1 July 2011, Institute of Electrical and Electronics Engineers, 2011.

[17] Martin P. and Salaun E.: *Generalized Multiplicative Extended Kalman Filter for Aided Attitude and Heading Reference System*. 2010 AIAA Guidance, Navigation and Control Conference, Toronto, Ontario, Canada, 2–5 Aug. 2010, American Institute of Aeronautics and Astronautics, Reston 2010.

[18] Roan P., Deshpande N., Wang Y. and Pitzer B.: *Manipulator State Estimation with Low Cost Accelerometers and Gyroscopes*. 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems, Vilamoura, Algarve, Portugal, 7–12 Oct. 2012, Institute of Electrical and Electronics Engineers, 2012, pp. 4822–4827.

[19] Sun M., Cao X., Song M., Izadi S., Benko H., Guimbretiere F., Ren X. and Hinckley K.: *Enhancing Naturalness of Pen-and-Tablet Drawing through Context Sensing*. On-line: http://research.microsoft.com/en-us/people/xiangc/its2011_pentabletcontext.pdf, last visited on 06 Jan. 2014.

[20] Sun G., Yong L., Xie J., Garratt M. and Wang C.: *Implementing quaternion based AHRS on a MEMS multisensor hardware platform*. IGNSS Symposium 2013, Outrigger Gold Coast, Qld Australia, 16–18 July 2013, International Global Navigation Satellite Systems Society, Tweed Heads 2013.

[21] Tuononen A.: *Optical position detection to measure tyre carcass deflections and implementation for vehicle state estimation*. Doctoral dissertation, Helsinki University of Technology, TKK Dissertations 205. Espoo 2009.