USING GENETIC ALGORITHM IN MAKING OPTIMAL CONTROL DECISIONS

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Summary

The problems presented in this article deal with operation process control in complex systems of technological object operation. Making appropriate control decisions directly influences the possibility of correct and effective carrying out of tasks assigned to the system. The paper presents a method of determining the optimal strategy for control of technological object operation process on the basis of genetic algorithm. In the presented method, determining of optimal strategy for control of technological object operation process involves the choice of a sequence of control decisions made in individual states of the modeled operation process. The method involves a choice, out of the possible decision options, of the best strategy for operation process control for which the function constituting the evaluation criterion has extreme value. Depending on one's needs, the genetic algorithm including the obtained model of operation process may be implemented for mathematic formulation and solution of a wide array of problems connected with control of complex systems of technological object operation. It pertains mostly to the economic analysis, risk management and safety management of complex systems of technological object operation. The paper presents an example of determining optimal strategy for control (decision sequence) when the criterion function includes availability of means of transport used in a selected operation system.

Keywords: operation process, control decisions, genetic algorithm

1. Introduction

Correct and effective functioning of complex technological object operation systems is possible only when control decisions made by system decision makers are rational. In systems where a complex technological object operation process is carried out, the choice of rational control decisions from among the possible decision options is a difficult and complicated issue. In virtual complex systems of technological object operation, decision making process should be carried out with the use of appropriate methods and mathematic tools, not in an 'intuitive' way, based only on the knowledge and experience of the system decision makers. Using appropriate mathematic methods for operation process control facilitates the choice of rational control decisions in a way that assures correct and effective carrying out of tasks assigned to the system.

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In the case of complex technological object operation systems, in order to determine the optimal strategy for operation process control it is necessary to implement appropriate and effective methods and mathematic tools. The paper presents genetic algorithm as an example of a tool supporting the process of determining the optimal control strategy.

In technical literature one may find numerous studies dealing with the theoretical description as well as examples of practical uses of genetic algorithm in searching for optimal solution, i.e.: [1, 3, 9, 10, 12, 13, 14]. The genetic algorithm belongs to the group of non-determinist methods of determining the optimal solution in which particular solutions are random modifications of previous solutions and are parts of them in a significant manner. The basic assumption for using genetic algorithm in searching for an optimal solution is the fact taken from evolution theory claiming that the greatest probability of modification involves solutions with the greatest degree of adaptability defined by the fitness function (optimization task objective function).

Genetic algorithm may serve as a convenient tool the implementation of which facilitates the use of a complicated rational control decision making process in complex technological object operation system.

2. Description of genetic algorithm operation

For the description of the operation of genetic algorithm presented in the article, terminology generally used in technical literature was used, while, at the same time reflecting names and terms connected with optimal strategy for technological object operation process:

- gene (decision) – individual element of the chromosome – while looking for optimal strategy, defined by a specific decision made in a given decision-making state of the analyzed operation process;
- chromosome (strategy) – an object representing crucial variables in the process of looking for optimal solution (eg. optimal strategy). It consists of an ordered gene sequence (decision) and constitutes an encoded form of possible solutions (permissible strategies);
- population – chromosome set (strategy set). Population size is preliminary established and remain constant during the calculation procedure. During the operation of the algorithm elements of the population (chromosomes) undergo modification according to the preliminary accepted scheme so that, after modification, they maintain certain features of the elements (chromosomes) from earlier populations and as a result of the operation of the random factor;
- fitness function – objective function or a function connected with objective function in the process of looking for optimal solution (optimal strategy). It makes a numerical evaluation of the adaptability of individual elements (strategies) possible.

Figure 1 presents a general chart of the operation of the genetic algorithm in the case of determining optimal strategy δ^* for control of technological object operation process.

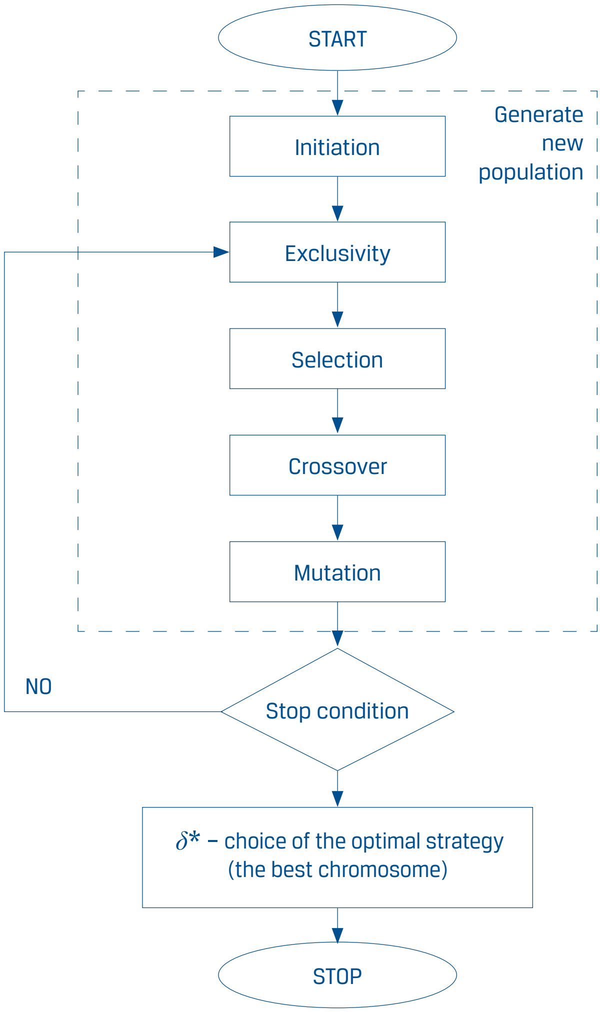

Fig. 1. Scheme of the genetic algorithm of choice of the optimal strategy δ^*

The following stages of genetic algorithm operation are presented below:

Stage I – INITIATION

Initiation is an initial stage of the carrying out of genetic algorithm. At this stage, establishing of the basic parameters of the algorithm and of the rules of encoding of optimization variables, determining of initial population, outlining fitness functions as well as determining the value of fitness function value for individual chromosomes (δ strategies) of initial population all take place.

Stage I.a – Determining basic parameters of genetic algorithm

The basic parameters of genetic algorithm are as follows:

- length m of the chromosome, determined by the number of genes in the chromosome. The number of genes in the chromosome equals the number of the analyzed crucial variables in the given optimization task;
- population size n, or the number of chromosomes in the population. Due to the precision and credibility of results, an appropriate choice of the number of chromosomes in the population is an important problem. The number of chromosomes in the population should not be too small, as it limits the possibility of population evolution, and, as a result, the analyzed (searched) solution subset becomes narrowed down in successive iterations (for successive populations). On the other hand, a very large population may cause significant prolongation of calculation periods in successive iterations. The number of chromosomes in the population depends on both the length of chromosome and the implemented encoding method;
- η factor – determining the probability of selecting chromosomes on the basis of elitism. The principle of elitism pertains to the choice of the best adapted chromosomes belonging to the previous population and copying them for the new population. Depending on the size of the created new population, one or several best adapted are copied from previous population;
- κ factor – determining the probability of hybridization. Hybridization involves exchange of genes between chromosomes coming from individual parent pairs. As a result of the hybridization operation, the chromosomes of the offspring are created, a combination of genes of respective pairs of parent chromosomes;
- μ factor – determining the probability of mutation. Mutation is the final stage of chromosome generation in the new population and pertains to the change of individual genes of the chromosome of the offspring created at the stage of hybridization completely at random.

Stage I.b – Determining optimization variable encoding rules

While choosing the encoding method, several basic rules should be taken into account:

- using a chosen encoding method one needs to take into consideration that it only makes it possible to encode a finite number of elements from the acceptable solutions (chromosomes) set, determined by the number of possible chromosomes and their length (the number of genes in a chromosome);
- the chosen method must unequivocally make it possible to identify the individual elements of the acceptable solutions (chromosomes) set;
- the chosen method should eliminate the possibility of creating, as a result of genetic algorithm operation, a chromosome without an equivalent in the set of acceptable solutions.

While using genetic algorithm, from among the numerous methods of optimization variables encoding, binary encoding is the one used the most often. The binary encoding method makes it possible to encode individual elements of acceptable solutions (chromosomes) so that the number of genes in a chromosome equals the number of analyzed crucial optimization variables. When using binary encoding method the establishing of the number of m chromosome genes unequivocally determines the maximum number of chromosomes in the set of acceptable solutions, which amounts to 2^m.

Stage I.c – Determining initial (starting) population

Having established basic optimization parameters as well as encoding rules, with the help of random method, the initial population $\Delta^{(0)}$ with n number of elements (chromosomes) is generated in the form of the following sequence

$$\Delta^{(0)} = \left\{\delta_1^{(0)}, \delta_2^{(0)}, \ldots, \delta_{n-1}^{(0)}, \delta_n^{(0)}\right\}. \tag{1}$$

The order of drawn elements (chromosomes) in the population is arbitrary and there is a possibility of multiple occurrence of the same elements (chromosomes) in the same population.

Stage I.d – Determining the value of fitness function for initial population

Optimization of a given task with the use of genetic algorithm is possible only when we have the so-called fitness function at our disposal, for which it is possible to find maximum and minimum values (depending on the analyzed optimization task). A fitness function may be both an objective function and any function strictly connected with the objective function of the analyzed optimization task. Determining the value of fitness function makes it possible to numerically evaluate the adaptability of individual chromosomes in the analyzed population.

If in a given optimization task:

- the objective function f_C is defined in set X, or $f_C : X \to R$,
- $\delta(x)$ means a chromosome unequivocally identifying element $x \in X$,
- – a set of acceptable solutions Y is determined, being a subset of set X.

Then the function $f_P : \Delta \to R$ is a fitness function of the analyzed optimization task, where Δ means a chromosome set of m length, consisting of binary genes $d_j \in \{0,1\}, \ j = 1,2,\ldots,m$, defined as follows:

$$\Delta = \{\delta(x) : x \in Y\}, \tag{2}$$

where:

$$\delta(x) = [d_1, d_2, \ldots, d_m] : d_j \in \{0,1\}, \ \ j = 1,2,\ldots,m \ . \tag{3}$$

The fitness function $f_P : \Delta \to R$ thus determined may be an equivalent of the objective function $f_C : X \to R$ in the following form:

- in the case of objective function maximization

$$f_P(\delta(x)) = f_C(x), \tag{4}$$

- in the case of objective function minimization

$$f_P(\delta(x)) = -f_C(x) \, . \tag{5}$$

The value of fitness function $f_P(\delta(x))$ may be determined when the relation between chromosome set Δ and acceptable solution set Y as well as the relation between fitness function $f_P(\delta(x))$ and the objective function $f_C(x)$ are known.

Stage II – GENERATING POPULATIONS IN SUCCESSIVE ITERATIONS

The basic assumption in optimization method with the use of genetic algorithm is for the chromosome populations generated in successive iterations to be better adapted than chromosome populations generated in previous iterations. It means that the determined values of fitness function $f_P(\delta(x))$, for successive generated chromosomes are larger and larger when looking for maximum and smaller and smaller when looking for minimum.

The result of carrying out of this stage is generation of elements (chromosomes) of new populations $\Delta^{(I)}$ (created in successive iterations I) out of elements (chromosomes) of previous populations $\Delta^{(I-1)}$. The method of generating elements of new populations involves n–fold drawing of n chromosome pairs, the so-called parent pair, and then creating n offspring of the new population as a result of n–fold use of successive operations: selection, hybridization and mutation. As a result of hybridization and mutation operations, the selected chromosomes are suspect to random modifications which may cause the risk of losing the best adapted chromosome from the previous population. Because of that, the so-called elitism principle is used in practice.

Stage II.a – Elitism

According to elitism principle at least one form among the best adapted chromosomes is copied for the new population. When using genetic algorithm, in order to assure a better and better adaptability of successive populations one assumes that the greatest influence over the new populations should be exerted by elements (chromosomes) belonging to previous population for which fitness functions asserted the highest values. According to this assumption, weakly adapted chromosomes should not end up in the newly created population.

Elitism principle involves the choice of the best (best adapted) elements (chromosomes) from out of the elements (chromosomes) of previous population and copying them to new populations. The number of copied chromosomes may differ. Most often it is assumed that one or more of the best adapted chromosomes are copied from previous population. Elitism principle makes a better functioning of genetic algorithm possible and involves the fact that the best result (the best adapted chromosome) is remembered (copied to successive populations) as long as a better solution is found – a chromosome, for which the value of the function is higher.

For the choice of elements to be copied to new populations, random methods are often used. Then, a value for the η –factor is determined, defining the probability of the choice of chromosomes based on elitism principle. It means that the chromosomes to be placed in the new population are chosen according to the rule of elitism with η probability, while choosing based on the rule of selection, hybridization and mutation with $1 - \eta$ probability.

Stage II.b – Selection

The goal for the stage of selection is the choice of such chromosomes from out of the chromosomes of the previous population, which at the stage of hybridization will create the so called parent chromosome pairs. Chromosome selection is a random process in which the choice of parent pair should be significantly influenced by the given chromosome having desirable features (fitness function value). It means that if a given chromosome is assigned a better (higher or smaller, respectively) value of fitness function, the probability of drawing this chromosome for the parent pair should be higher. One of the simplest and most often used drawing methods at this stage is the so-called roulette wheel method which meets the condition of proportionality of the chance to draw a given chromosome for the value of fitness function assigned to it.

Stage II.c – Hybridization

The objective of hybridization operation is transferring of the features of individual parent chromosome pairs (chosen at selection stage) to the offspring chromosomes, newly created at the stage of hybridization. Hybridization operation involves an exchange, with the help of specific hybridization operator, of genes between chromosomes coming from individual parent pairs. As a result of carrying out of hybridization operation, offspring chromosomes are created as certain combinations of genes of appropriate parent chromosome pairs.

In order to ascertain whether hybridization operation is implemented, the value of the factor is set at $\kappa \in \langle 0,1 \rangle$, determining the probability of hybridization taking place. Then, for each of the parent chromosome pairs (from previous population) the number $x \in \langle 0,1 \rangle$ is drawn. Hybridization operation, for the given parent chromosome pair, is carried out when the value of the drawn number $x < \kappa$. Nevertheless, when $x \geq \kappa$, hybridization is not carried out and one of the parent chromosomes (selected by drawing) is copied to the successive population. It is worth noticing that for $\kappa = 0$ hybridization is never carried out, while for $\kappa = 1$ it is carried out very often (hybridization will be carried out for the majority of parent chromosome pairs).

The correct choice of the method of hybridization influences the correctness of the operation of genetic algorithm. Unfortunately, there is no single superior way of chromosome hybridization while the effectiveness of its selection depends on the analyzed objective function of the given optimization task. Many hybridization methods have been worked out, used in optimization with the use of genetic algorithm. Among the many available hybridization operators, the ones most often used in practice are:

- one point hybridization operator,
- two point hybridization operator,
- homogenous hybridization operator,
- AND hybridization operator,
- XOR hybridization operator,
- BLX-α hybridization operator,

- MIN-MAX hybridization operator,
- FCB hybridization operator.

Stage II.d – Mutation

Mutation is the final stage of generating elements (chromosomes) of the new population and involves the change of individual genes of the offspring chromosome created previously at the hybridization stage. Using mutation makes it possible to include among the elements of the new population also these chromosomes which, by definition, are practically impossible to obtain as a result of hybridization only (out of the elements of previous population).

At the stage of mutation the changes of individual genes are carried out in a completely random way. The values of the probability of the occurrence of mutation for individual chromosome genes are determined at the stage of the initiation of genetic algorithm. In order to tell whether the mutation operation is to be implemented, the value of the factor is set at $\mu \in \langle 0,1 \rangle$, determining the probability of mutation occurrence. Then, for each d_j gene of the analyzed chromosome (created at the stage of hybridization) the number $x_j \in \langle 0,1 \rangle$ is drawn. Mutation of d_j gene is carried out when the value of the drawn number $x_j < \mu$. Nevertheless, when $x_j \geq \mu$, mutation of d_j gene is not carried out. It is worth noticing that for $\mu = 0$, hybridization is never carried out, while for $\mu = 1$ it is carried out very often (mutation will be carried out for majority of genes).

Stage III – STOP CONDITION

When the choice of optimal strategy (superior chromosome) is made on the basis of genetic algorithm, it is possible to implement two stop conditions:

- Attaining the assumed number of iterations,
- Small changes of the value of objective function (fitness function) determined for the strategy (chromosome) best adapted among the elements of the tested populations within the course of successive iterations.

3. The choice of the optimal control strategy for the operation process of technical objects

Due to the random nature of the factors influencing the running of the technical objects (transport means) operation process introduced in a complex system, most often in the process mathematical modelling of the operation process, stochastic processes are used. Random process includes a wide implementation of Markov and semi-Markov process for modelling the operation process for technical objects, whereas in the case of the issues involving control of operation processes, decision-making Markov and semi-Markov processes are used [2, 4, 5, 6, 7, 8, 11].

Assuming that the analyzed model of technical object operation process is a random process $\{X(t): t \geq 0\}$ of finite number of process states i = 1,2,...,m, then:

$$D_i = \left\{ d_i^{(1)}(t_n), d_i^{(2)}(t_n), \ldots, d_i^{(k)}(t_n) \right\}, \tag{6}$$

means a set of all possible control decisions which can be implemented in i-state of the process at the moment of t_n, where $d_i^{(k)}(t_n)$ means k-control decision made in i-state of the process, at the moment of t_n.

In the case of optimization task involving the choice of optimal strategy of technical object operation process control from among the acceptable strategies, then as the strategy we understand the δ sequence, where the words are the vectors, comprising of the decision $d_i^{(k)}(t_n)$ made in the following moments of the t_n changes of the state of the process $X(t)$:

$$\delta = \left\{ \left[d_1^{(k)}(t_n), d_2^{(k)}(t_n), \ldots, d_m^{(k)}(t_n) \right] : \ n = 0,1,2,\ldots \right\}. \tag{7}$$

The choice of appropriate control strategy δ called the optimal strategy δ, concerns the situation, when the function representing the selection criterion of the optimal strategy takes an extreme value (minumum or maximum).

In order to determine the optimal control strategy (decision sequence) it is possible to implement decision-making semi-Markov processes. The decisive semi-Markov process is a stochastic process $\{X(t): t \geq 0\}$, the implementation of which depends on the decisions made at the beginning of the process t_0 and at the moments of changing the process t_1, t_2, ..., tn, ... In case of implementation of the decisive semi-Markov processes making the decision at the moment of t_n, k-controlling decision in i-state of the process means a choice of i-verse of the core of the matrix from the following set:

$$\left\{ Q_{ij}^{(k)}(t) : t \geq 0, \ \ d_i^{(k)}(t_n) \in D_i, \ \ i, j \in S \right\} \ , \tag{8}$$

where:

$$Q_{ij}^{(k)}(t) = p_{ij}^{(k)} \cdot F_{ij}^{(k)}(t). \tag{9}$$

The choice of the i-verse of the core of the process specifies the probabilistic mechanism of evolution of the process in the period of time ‹t_n; t_{n+1}). This means that for the semi-Markov process, in case of the change of the state of the process from one into i-one (entry to the i-state of the process) at the moment t_n, the decision is made $d_i^{(k)}(t_n) \in D_i$ and according to the schedule $\left(p_{ij}^{(k)} : j \in S \right)$ j-state of the process is generated, which is entered at the moment of t_{n+1}. At the same time, in accordance with the schedule specified by the distributor $F_{ij}^{(k)}(t)$, the length of the period of time is generated ‹tn; t_{n+1}) to leave the i-state of the process, when the next state is the j-state.

In case of the implementation of the genetic algorithm to determine the optimal strategy of controlling the operation processes for technical objects, the following guidelines should be considered:

- the examined stochastic process is the m-state decisive semi-Markov process,
- in each state it is possible to implement one of the two ways of performance (called a decision),
- if the decisions are marked as 0 and 1 then the number of control strategies to be implemented for the m-state model of the operation process of the means of transport amounts to 2^m,
- the set of control strategies is the set of functions:

$$\delta : S \rightarrow D \, , \tag{10}$$

where:

S – is the set of the states of the process, $S = \{1, 2, \ldots, m\}$,

D – is the set of decisions made in the states of the process, $D = \{0,1\}$.

On the basis of the following guidelines each possible control strategy can be presented as m-positioning sequence consisting of 0 and 1. This is then the positioning binary number. Therefore, an exemplary control strategy for the model of the operation process consisting of m = 9 states can be determined in the following way $\delta = [1,0,1,1,0,1,0,0,1]$.

4. Example of determining optimal technical object operation process control strategies

Presented below is an example of assigning optimal operation process control strategy carried out in the chosen transport means operation system – the city bus transport system. In the presented example, the criterion of determining optimal strategy δ^* constitutes the value of the function describing the availability of the technical object (transport means) G^{OT}. The choice of the optimal strategy δ^* is made on the basis of the following criterion:

$$G^{OT}\left(\delta^*\right) = \max_{\delta}\left[G^{OT}(\delta)\right] \, . \tag{11}$$

The evaluation of the availability of transport means may be performed on the basis of mathematical model of the operation process carried out in the tested system of transport means (city buses) operation.

Due to the identification of the analyzed operation process of transport means, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation process states, shown in Figure 2.

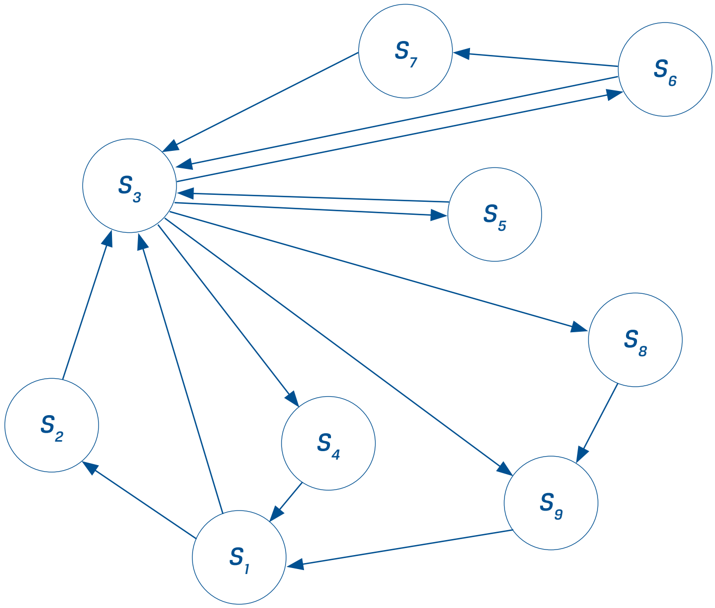

Fig. 2. Directed graph representing the transport means operation process

S_1 – refueling, S_2 – awaiting the carrying out of the task at the bus depot parking space, S_3 – carrying out of the transport task, S_4 – awaiting the carrying out of the task between transport peak hours, S_5 – repair by technical support unit without losing a ride, S_6 – repair by the emergency service with losing a ride, S_7 – awaiting the start of task realization after technical support repair, S_8 – repair in the serviceability assurance subsystem, S_9 – maintenance check on the operation day

The mathematical model of the analyzed transport means operation process was built with the use of the semi-Markov processes theory. The semi-Markov $X(t)$ process is one, where periods of time between the changes of consecutive process states have arbitrary probability distributions and a transfer to the consecutive state depends on the current process state. Using the semi-Markov processes in mathematical modelling of the operation process, the following assumptions were put forward:

- the modelled operation process has a finite number of states S_i, i = 1,2,...,9,
- if technological object at moment t is in state S_i, then $X(t) = i$, where i = 1,2,...,9,
- the random process $X(t)$ being the mathematical model of the operation process is a homogenous process,
- at moment t = 0, the process finds is in state S_3 (the initial state is state S_3), i.e. $P\{X(0) = 3\} = 1$.

The homogenous semi-Markov process is unequivocally defined when initial distribution and its kernel are given. Form our assumptions and based on the directed graph shown in Figure 2, the initial distribution $p_i(0)$, i = 1,2,...,9 takes up the following form:

$$p_i(0)=\begin{cases}1 & gdy \quad i=3\\ 0 & gdy \quad i\neq 3\end{cases}, \tag{12}$$

where:

$$p_i(0)=P\{X(0)=i\}, \quad i=1,2,\ldots,9\ ; \tag{13}$$

whereas the kernel of process $Q(t)$ takes up the form:

$$Q(t)=\begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t)\\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0\\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t)\\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

where:

$$Q_{ij}(t)=P\{X(\tau_{n+1})=j, \tau_{n+1}-\tau_n\leq t \,|\, X(\tau_n)=i\}, \quad i,j=1,2,\ldots,9\ , \tag{15}$$

$$Q_{ij}(t)=p_{ij}\cdot F_{ij}(t), \tag{16}$$

$$p_{ij}(t)=P\{X(t)=j \,|\, X(0)=i\} \tag{17}$$

means that the conditional probability of transfer from state S_i to state S_j,

$$F_{ij}(t)=P\{\tau_{n+1}-\tau_n\leq t \,|\, X(\tau_n)=i, X(\tau_{n+1})=j\}, \quad i,j=1,2,\ldots,9 \tag{18}$$

is a distribution function of random variable signifying period of duration of state S_i, under the condition that the next state will be state S_j.

In order to assign the values of limit probabilities p_i* of staying in the states of semi-Markov model of transport means operation, the following were created: matrix P of the states change probabilities and matrix Θ of conditional periods of duration of the states in process $X(t)$:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{19}$$

$$\Theta = \begin{bmatrix} 0 & \overline{\Theta}_{12} & \overline{\Theta}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Theta}_{34} & \overline{\Theta}_{35} & \overline{\Theta}_{36} & 0 & \overline{\Theta}_{38} & \overline{\Theta}_{39} \\ \overline{\Theta}_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{63} & 0 & 0 & 0 & \overline{\Theta}_{67} & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\Theta}_{89} \\ \overline{\Theta}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{20}$$

Based on the matrix P and on the matrix Θ, average values $\overline{\Theta_i}$ of non-conditional duration periods of process states were defined, according to the dependence:

$$\overline{\Theta_i} = \sum_{i=1}^{9} p_{ij} \cdot \overline{\Theta}_{ij}, \quad i, j = 1,2,...,9 . \tag{21}$$

Therefore, the boundary probability p_i* for staining in the states of the semi-Markov processes can be determined on the basis of the boundary statement for the semi-Markov process [2], in accordance with the following pattern:

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)}, \tag{22}$$

where probabilities $\pi_i,\ i \in S$ constitute the stationary layout of the implemented Markov's chain in the process which fulfils the system of linear equations:

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1. \tag{23}$$

Availability of a single technical object defined on the basis of the semi-Markovian model of operational process is determined as the sum of limit probabilities p_i* of remaining at states belonging to the availability states set:

$$G^{OT} = \sum_i p_i^*, \quad dla \quad S_i \in S_G . \tag{24}$$

In order to define availability of technical objects (means of transport) based on the semi-Markovian model of operational process, the operational states of the technical object should be divided into availability states S_G and non-availability states S_{NG} of the object for the carrying out of the assigned task. In the presented model, the following technical object availability states were enummerated:

- state S_2 – awaiting the carrying out of the task at the bus depot parking space,
- state S_3 – carrying out of the transport task,
- state S_4 – awaiting the carrying out of the task between transport peak hours,
- state S_7 – awaiting the start of task realization after technical support rep air.

Then, with the use of the MATHEMATICA software, the limit probability p_i* of staying in states of semi-Markov process and the availability of technical objects of the transport system were determined:

$$G^{OT} = \frac{p_{12} \cdot (p_{34} + p_{38} + p_{39}) \cdot \overline{\Theta_2} + \overline{\Theta_3} + p_{34} \cdot \overline{\Theta_4} + p_{35} \cdot \overline{\Theta_5} + p_{36} \cdot p_{67} \cdot \overline{\Theta_7}}{\left[(p_{34} + p_{38} + p_{39}) \cdot \left(\overline{\Theta_1} + p_{12} \cdot \overline{\Theta_2}\right)\right] + \overline{\Theta_3} + p_{34} \cdot \overline{\Theta_4} + p_{35} \cdot \overline{\Theta_5} + \left[p_{36} \cdot \left(\overline{\Theta_6} + p_{67} \cdot \overline{\Theta_7}\right)\right] + p_{38} \cdot \overline{\Theta_8} + (p_{38} + p_{39}) \cdot \overline{\Theta_9}} . \tag{25}$$

For the analyzed model of transport means operation process the values of genetic algorithm input parameters and possible decisions made in decision-making process states were determined (Table 1) as well as, based on operational data, absolute values of process state duration periods were defined (Table 2).

Genetic algorithm input parameter values:

a) Length of chromosome m = 9
b) Size of population n = 100
c) Number of iterations I = 100
d) Probability of chromosome selection via elitism principle η = 0.2
e) Probability of hybridization occurrence κ = 1
f) Probability of mutation occurrence μ = 0.05

Table. 1. The control decisions in the states of the analyzed operation process

Process state	Decision „0" – $d_i^{(0)}$	Decision „1" – $d_i^{(1)}$
S_3	The route marked code L („light" conditions of the delivery task)	The route marked code D („difficult" conditions of the delivery task)
S_5	Treatment by a PT type B (basic range)	Treatment by a PT type E (extended range)

Table. 1. The control decisions in the states of the analyzed operation process (cont.)

S_6	Treatment by a PT type B (basic range)	Treatment by a PT type E (extended range)
S_8	Treatment in positions PZZ type N (normal)	Treatment in positions PZZ type I (intensive)
S_9	Operate in positions OC type N (normal)	Operate in positions OC type I (intensive)

Table. 2. Code markings of decisions and absolute process state duration periods

Process state	$d_i^{(0)}$	$d_i^{(1)}$	$\Theta_i^{(0)]}$ [h]	$\Theta_i^{(0)]}$ [h]
S_1	0	1	0.096	0.096
S_2	0	1	5.659	5.659
S_3	0	1	8.852	7.967
S_4	0	1	3.450	3.450
S_5	0	1	0.070	0.063
S_6	0	1	0.545	0.436
S_7	0	1	0.442	0.442
S_8	0	1	3.744	2.995
S_9	0	1	0.122	0.092

Next, calculations were made with the help of developed computer software, implemented genetic algorithm, written at: *R Development Core Team (2011). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.* As a result of the calculations performed, for the adopted criterion (11) the optimal operation process control strategy was determined in the tested system of transport means operation – city bus operation system. Calculation results were presented in Table 3.

Table. 3. Optimal transport means operation process control strategy and criterion function value, determined on the basis of genetic algorithm (for hypothetical operational data)

Optimal strategy $\delta*$	$G^{OT}(\delta*)$
[1,1,1,0,0,1,0,0,1]	0.8426

5. Summary

The presented method determining the optimal strategy controlling, means determination a sequence of control decisions made in individual states of the model process, for which the function constituting the criterion of evaluation achieves an extreme value. In order to specify the optimal strategy controlling the operation process of the technical objects the genetic algorithm was recommended. Due to the general character the presented method canoe implemented for solving a broad spectrum of optimization issues concerning the exploitation systems for the technical objects such as: controlling availability and reliability, analysis of costs and profits, analysis of risk and safety etc.. In each case there is a necessity to form properly the definition of the criterion and specifying possible control decisions made in the states of the examined operation process of the technical objects.

The method of determining optimal technical object operation process control strategy with the use of genetic algorithm presented in the article, constitutes one of the stages of work the goal of which is developing a complex method of technical object operation process control with the use of decision-making models. A complex method of transport means operation process control is supposed to make it possible to control both the processes carried out at the executive subsystem (evaluation of transport means tasks carried out) as well as at the serviceability assurance subsystem (evaluation of service and repair tasks carried out), taking into consideration both the technical and economic criteria of the functioning of this type of operation systems.

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