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STUDY OF THE PROBLEM AND THE IDEA OF OPERATION OF FOUR-WHEEL-STEERING CARS

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Abstract

The article takes the form of a review paper and focuses on the control problems of four wheel steering vehicles. The main issues related to the control of four wheel steering (4WS) vehicles, the historical development, the current state and the approach to the construction of 4WS vehicles have been discussed. The whole of theoretical issues related to the control of the car direction (including elements related to mathematical modelling and simulation studies of the vehicle motion, issues of synthesis and analysis of the 4WS control algorithms) were presented systemically. The presented collection of literature is based on a rather extensive review relating to 4WS cars and on considerations shaping the concept of development of the topic of control algorithms in 4WS cars.

A preliminary concept of control in autonomous cars is also presented and will be explain what type of road manoeuvres it is going to analyse. The authors have also shown an example of the computational apparatus used.

Keywords: autonomous vehicle; control; development; four wheel steering cars

1. Introduction

The steering system is a fundamental component of any wheeled vehicle which operates on the road by controlled rotation of road wheels mounted on steering knuckles. The classic and most common car steering systems refer to the 2WS (two wheel steering) structure in which a turn of the steering wheel results in a turn of the mechanically coupled two front wheels. However, for a long time automobile constructors have also worked on steering system solutions in a 4WS (four wheel steering) structure, in which the steering involves not only the front wheels but also the rear wheels [14, 22, 47].

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As it is known from theory and practice, steering by turning the rear wheels makes it easier to manoeuvre a slowly moving vehicle (e.g. a forklift) in a small space. Unfortunately, at higher speeds typical of road traffic, when tyre drift starts to count, such steering is extremely difficult due to the then exceptional oversteer of the vehicle and the threat of instability. This is why the first (still in the interwar period) automotive applications of the 4WS structure concerned only slow-moving trucks (e.g. Nash Quad) and off-road vehicles (e.g. Mercedes 170 VL). In these vehicles, the steering mechanism caused the rear wheels to turn in the opposite direction to that of the front wheels, which provided a significant reduction in the turning radius at low speed, thus increasing manoeuvrability.

However, research carried out in many centres on the active safety and driveability of cars travelling at high speeds has indicated that it would be beneficial to stabilise their distorted movement if the rear wheels were angled slightly in line with the steering direction of the front wheels. This postulate of understeer (contradictory to the concept of rear-wheel steering improving vehicle manoeuvrability and increasing oversteer) gave rise to the development of self-stabilising mechanisms as elements of rear-wheel suspension. Such passive mechanisms based on elastokinematic and multi-link systems appeared at the beginning of the 1980s in high-end passenger cars (e.g. Weissach mechanism in Porsche 928).

It is worth mentioning that already in the late eighties, active self-stabilising mechanisms were introduced, equipped with actuators correcting the angular position of the rear wheels when cornering at high speed (e.g. HICAS - High Capacity Actively Controlled Suspension in the Nissan Skyline). The concept of active stabilisers as part of the suspension system (rather than a steering system with wheels mounted knuckles) is still being developed and used in many recent designs. Representative of this is the AGCS (Active Geometry Control Suspension) used on the Hyundai Sonata. In this system

a two-armed lever set in a rotary position and moved by an electric actuator to move the control arms in the direction of the rear wheels.

Steering systems with a 4WS structure for passenger cars, i.e. those travelling at a wide range of speeds, have been developed since the early 1980s, when special rather complex mechanisms were developed to adapt the steering control of the rear wheels to the travelling speed. According to the general idea of 4WS control expressed in Figure 1, at low driving speeds (e.g. when manoeuvring in a car park) the steering direction of the rear wheels is opposite to that of the front wheels, while at high speeds the steering directions of the front and rear wheels are compatible. The relation between rear wheel steering angle and front wheel steering angle results from the adopted (here simplified by linear interval approximation) characteristic which makes the ratio dependent on the driving speed. At a certain characteristic speed V_0 (selected for a given vehicle and, as it turns out, different in each solution) the steering of the rear wheels is zero. In this way, the idea of the steering system in the 4WS structure meets the postulate of active safety (some understeer and stabilisation during road driving) and steering comfort (some oversteer and easier manoeuvring in a car park).

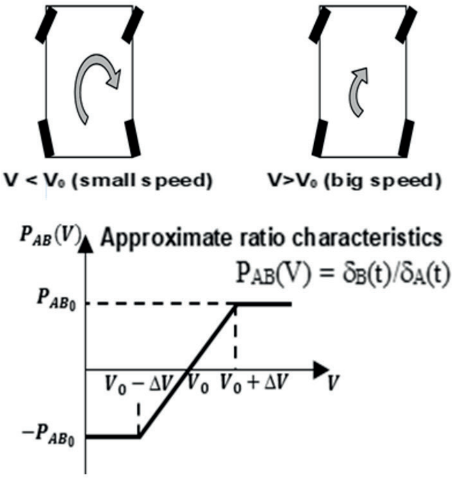

Fig. 1. Overall idea of steering front and rear wheels in 4WS vehicles [72]
Notation: P_{AB} – gear ratio between front and rear wheel twist angles (δ_A, δ_B)
V – vehicle speed, V_0 – characteristic speed when $\delta_B=0$

In the eighties, a number of passenger car designs were developed (mainly by Japanese companies), in which the idea of 4WS structure was applied in practice. In the nineties, a certain stagnation in the 4WS development could be observed due to the popularity of effective mechatronic systems increasing the comfort and active safety of a car steered in a 2WS structure (e.g. EPS - Electric Power Steering, ESP - Electronic Stability Program). However, in the last twenty years, with the introduction of advanced control techniques (digital controllers realizing by-wire complex control algorithms based on the motion measurements), the interest in the 4WS idea has been growing again. In many research centres, the search for more and more perfect 4WS control algorithms continues, which is reflected both in newly produced cars and in numerous scientific studies.

2. Development of 4WS vehicle design

The first patents relating to 4WS steering were filed by Honda engineers in the late 1970s. In 1981, a prototype was developed based on Honda Accord components, which had mechanically coupled front and rear steering. At speeds of less than 40 km/h and a steering angle of more than 240 degrees, the rear wheels turned in opposite directions to the front wheels. The rear wheels were able to turn 5 degrees when the steering angle of the front wheels reached 35 degrees. At speeds greater than 40 km/h and a steering wheel turn of less than 240 degrees, the rear wheels turned in the same direction as the front wheels. The twist of the rear wheels was ten times less than that of the front wheels. This was done entirely mechanically in a system that transferred control from front to rear (mechanism with shaft and planetary gear). The debut of the mechanical 4WS system developed at Honda took place in 1987 on the Prelude vehicle. With 4WS, the vehicle's turning radius was reduced by about 10% and vehicle stability increased. A shortcoming of the mechanical 4WS system was the perceived disturbance in the smoothness of its mechanisms. Therefore, the mechanical version of 4WS was not appreciated by users. In the next generations of the Honda Prelude, mechatronic elements appeared, which significantly improved the operation of the 4WS system. Newer generation models (1991 and 1997) introduced computer control and an electric motor instead of a shaft and planetary gearbox. This system was already similar to "modern" computerised 4WS systems. It is worth noting that the 4WS control system was also offered by Honda on request in other production models [14, 47].

Work on similar 4WS systems was also undertaken almost simultaneously by other Japanese companies, which patented their own versions of solutions. The idea of the Mazda developers was already presented in 1984 in the concept version of the MX model. Compared to the Honda design, this system was not only mechanical, but also had electronic and hydraulic components. The vehicle had speed and steering sensors. In this case, the shaft from the front to the rear steering mechanism only transmitted information about the steering angle to an electronically controlled hydraulic cylinder. At low speeds the rear wheels turned against the front wheels, but at high speed the rear wheels turned equally to the front wheels. This solution was introduced, for example, on the MX6 models from 1991. Mitsubishi also presented a similarly equipped Galant VR-4 as early as 1987 [47].

In this case, there was no mechanical connection between the front and rear steering mechanism, and the solution was electrohydraulic. Toyota had been working on the 4WS a little longer. Its first solution from 1989 was similar to Mazda's. However, in subsequent models, e.g. in the Celica Dual Mode 4WS completely new possibilities of adjusting the steering depending on the selected mode of operation (standard or sport) appeared. It is worth mentioning at this point that not all Japanese companies developed the 4WS concept by steering control of rear wheels embedded in steering knuckles. Nissan, for example, has since 1988 based its development on 2WS solutions, supplemented by the HICAS suspension systems mentioned in the first point, ensuring self-stabilisation at high speeds [14].

European and American car manufacturers have also worked on 4WS systems. This is particularly true in the last two decades, when mechatronic systems began to be implemented in vehicles, allowing not only the transfer of by-wire control to the rear wheels, but also its active shaping. A European example is the state-of-the-art 4Control integrated system developed by Renault for the Laguna GT and subsequently implemented in other vehicles of the brand. On the Laguna GT, the rear wheels turn in an opposite motion to the front wheels up to 60 km/h, and at higher speeds in the same motion. Their maximum steering angle is 3.5 degrees in the opposite direction and 2 degrees in the direction that the front wheels are turning. With this solution, the vehicle's turning diameter, even in the estate version, does not differ much from that of the smaller Clio model and is approximately 10.8 m. At the same time, this system reduces the steering angle. To turn the wheels one degree without 4Control, the steering wheel has to be turned 16 degrees, whereas with 4Control the steering wheel has to be turned 4 degrees less when the rear wheels are turned in the opposite direction to the front wheels. This improves response times and dramatically increases steering precision, especially when manoeuvring quickly. It should be noted that the introduction of 4WS control in the Laguna GT has modified its ESP stability system. The system module identifies the situation in which the vehicle brakes with different wheel adhesion. The controller adjusts the rear-wheel steering accordingly to maintain the driver's desired trajectory, which is done automatically. It is also worth mentioning that ESP starts later than in vehicles without 4WS. This improves the drivability of the vehicle [10].

A representative example of 4WS on the U.S.A. market is the Quadrasteer system by Delhi Automotive (the world leader in by-wire solutions), proposed in 2002 primarily for long pickups by General Motors (e.g. GMC Sierra Denali).

Contemporary 4WS steering systems from different manufacturers are technically different in detail, but conceptually similar (shown in Table 1).

Tab. 1. Characteristics of 4WS from different manufacturers [10]

Brand and type	Speed V_0 [km/h]	Steering angle [°]	Turning direction of the rear wheels in relation to the turning of the front wheels	Maximum rear wheel steering angle [°]
Honda Accord	≤40	>240	Against	5
		<240	Compliant	10 times less than the front wheels
	>40	>240		
		<240		
Mazda MX3	<35	-	Against	5
	=35		No turning of the rear wheels	0
	>35		Compliant	5
Nissan 300 ZX	<30		Against	7
	=30		No turning of the rear wheels	0
	>30		Compliant	7
BMW	<30		Against	2.5
	30, 50		No turning of the rear wheels	0
	>50		Compliant	7
Audi A8, Q7	<50		Against	5
	50, 80		No turning of the rear wheels	0
	>80		Compliant	2
Renault Laguna GT	<60		Against	3.5
	=60		No turning of the rear wheels	0
	>60		Compliant	3.5
Lexus	<80		Against	3
	=80		No turning of the rear wheels	0
	>80		Compliant	1.5
Porsche	<50		Against	3
	=50		No turning of the rear wheels	0
	>50		Compliant	1.5

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As you can see, with virtually every manufacturer, the so-called zero position of the 4WS system occurs at different speeds, and in the case of BMW, Audi A8 and Q7 there are even speed ranges. It is also worth noting that Lexus is one of the few that turns the rear wheels opposite to the front wheels up to 80 km/h, but to a maximum of three degrees, while the Nissan 300 ZX switches to driving stabilisation already above 30 km/h, but the wheels turn up to 7 degrees, which is the highest camber among all the manufacturers presented. Each brand, as you can see, has its own concept for the solution, as can be seen by the variation in speed when switching ratio. The steering angles of each manufacturer are also different, ranging from one and a half to 7 degrees.

In current solutions, everything is managed by a by-wire digital controller, and the actuator (or two actuators, separate for the right and left rear wheels, as in Porsche) are electric. The digital control in 4WS systems allows the operation of the rear steering mechanism to take into account not only the speed and angle of rotation of the steering wheel, but also, for example, the rotational speeds of the road wheels, the operation of ABS and ESP, lateral and longitudinal accelerations, etc.

Four wheel steering system, but already in a highly developed version and integrated with other systems, can be found in a number of, but not only luxury cars (Table 2).

Tab. 2. List of passenger cars 4WS [10]

Brand	Models
Audi	A8, Q7
Accura	RLX, TLX
BMW	7, 6, 5 Series
Cadillac	CT6
Chevrolet	Silverado
Efini	MS-9
Ferrari	GTC4Lusso, F12tdf
GMC	Sierra, Sierra Denali
Honda	Prelude, Accord, Ascot Innova
Infiniti	FX50 AWD, G35 Sedan / Coupe, J30t, M35, M45, Q45t
Lamborghini	Aventador S, Centenario, Urus, Huracán Evo
Lexus	GS, LC500
Mazda	626, 929, MX6, RX7, Eunos 800
Mercedes-Benz	Vito (model taxi)
Mitsubishi	Galant, 3000GT
Nissan	Cefiro, 180 SX, 240SX, 300ZX, Laurel, Fuga, Silvia, Skyline GTS, GTR
Porche	911 GT3/Turbo/Spyder, Cayenne, Panamera
Renault	Espace, Laguna, GT, Megane GT/RS, Talisman
Rolls-Royce	Culliman
Subaru	Alcyone SVX JDM
Toyota	Aristo, Camry, Carina, Corona , Celica , Soarer
Volkswagen	Touareg

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3. Overview of the theory related to the control of 4WS

The development of 4WS systems technology would not have been possible without the broad involvement of researchers working in the fields of analytical mechanics, dynamic systems, control theory and technology and mechatronics. Such a conclusion can be drawn from a review of numerous articles and conference papers on 4WS vehicles.

Initially, the researchers' attention was focused on strictly basic issues concerning the 4WS idea. The manoeuvrability of a car at low speeds is primarily a design problem related to kinematics and solved in design offices. Therefore, in the scientific literature we encounter few publications referring to these issues. The work [61], presenting kinematical relations for 4WS structures analogous to the well-known Ackerman relations for 2WS structures and the work [33] describing design with the use of analytical formulas and ADAMS and CATIA software are quite exceptional examples. On the other hand, the problems of the dynamics of 4WS motion at high speeds and the related control problems can be found in numerous publications.

In undertaking a study of the dynamics and control theory of 4WS vehicles, review works were initially sought. While there are many papers reporting on narrow sections of research, there are surprisingly few state-of-the-art or review articles. These include the widely cited article [12], which was however published over 30 years ago, and the more recent publications [24] and [21] (this work also deals with 2WS vehicles, but discusses modern concepts and control techniques in active steering systems very extensively). Reading the review papers confirms the conviction of how much progress in the design and research on 4WS vehicles has been made during this period.

In the study of the dynamics of 4WS vehicles, information derived from standard “open” steering tests (e.g. with stepped or sinusoidally varying steering wheel rotation forcing) is important in the first instance. The reported studies rarely deal with experiments on real vehicles, e.g. [43]. Most of the publications concerned simulation studies using models of considerable complexity, including nonlinear models with many degrees of freedom and many parameters. Examples are studies to analyse the drift process in 4WS vehicles [71] and to assess the influence of rear multi-link suspension solutions on the steerability of a 4WS vehicle [44, 45], or studies showing the significant influence of play and friction in the steering mechanisms (also with assistance on the trajectory of a steered vehicle in a 4WS structure [34] - Figure 2).

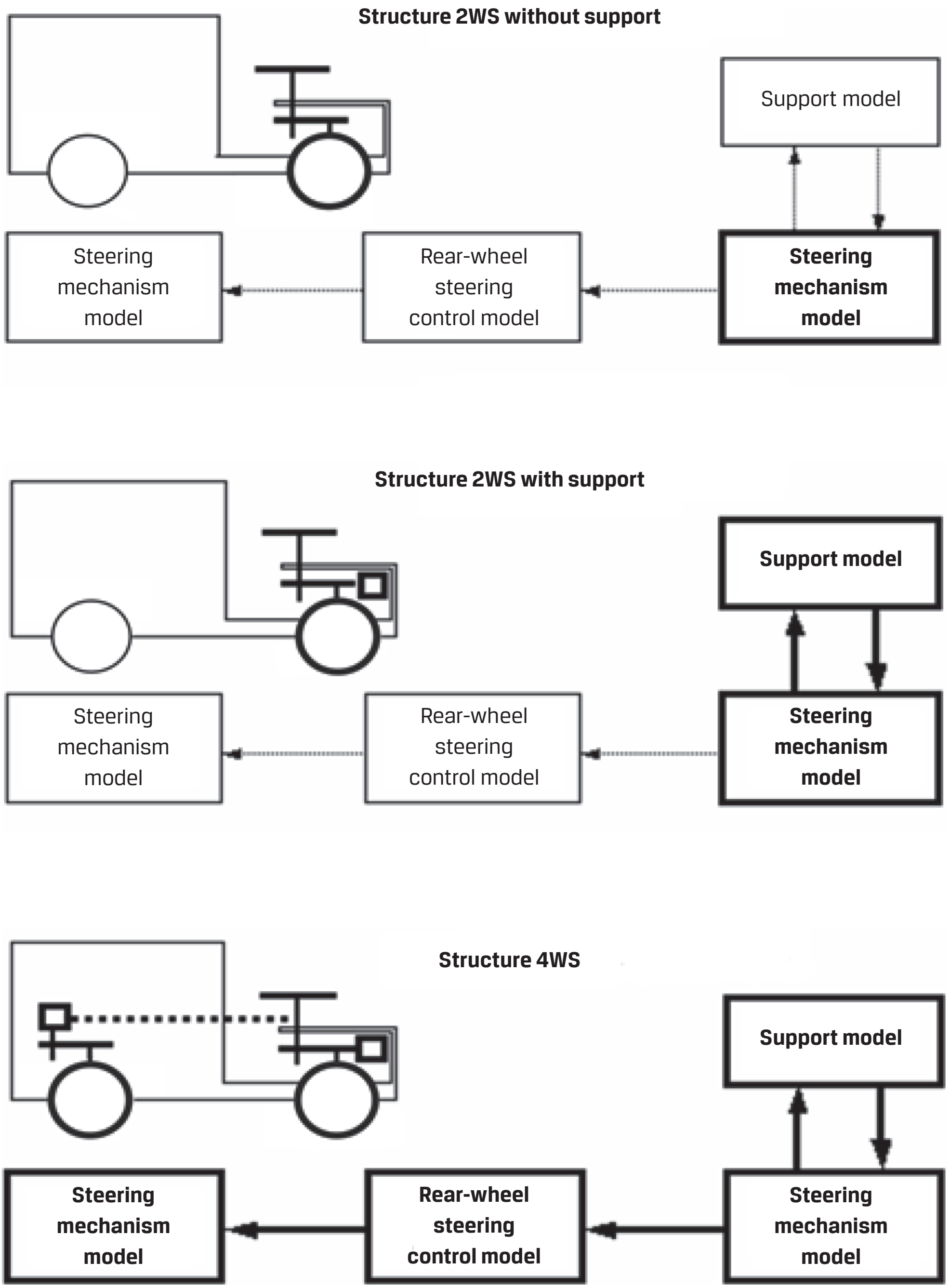

Fig. 2. Modelling concept in comparative studies of different steering structures in open tests [72]

Researchers have also undertaken simulation of „closed" (with driver) tests using complex mathematical models of the 4WS vehicle, e.g. [52].

In parallel with simulation studies, analytical studies were undertaken, also concerning the driver-vehicle 4WS-road system [16, 40, 53] on very simplified models (linear, two degrees of freedom bicycle type) using transmittance, operator calculus and frequency analysis. Such research is also in the context of the search for automatic steering systems [5]. It should be emphasised that the bicycle model treats a steered four-wheeled vehicle travelling at a fixed speed as a single-mass moving object on pneumatics, in which the reaction forces at the wheel-ride interface are related to equivalent points located on the longitudinal axis of the body block. In its mathematical formulation, there are only two variables expressing body motions (angular and linear) in the road plane and 7 parameters (vehicle mass and moment of inertia, wheel base and centre of mass position, travel speed, drift resistance coefficients resulting from tyre characteristics and road condition), which makes it possible to determine in the analyses the influence of the most important structural and operational factors on the time courses of control processes and their frequency characteristics. Such research made it possible to draw conclusions as to the influence of the 4WS transmission ratio characteristics on the phase delays in the transverse acceleration and yaw rate responses, on the oversteer and stability of motion and to formulate postulates as to the introduction into the system of steering control of rear wheels of regulators improving the dynamics of transverse motions of the vehicle, e.g. [4, 14, 18]. We can find interesting conclusions in [39], too.

Initially, the use of standard linear continuous regulators (even the simplest P regulator [45]) was considered in various executive configurations and measurement configurations generating control deviations. The introduction of classical regulation based on linear regulators mostly improved the lateral dynamics of the 4WS vehicle [26, 29, 30]. The articles [41, 60, 64] develop idea about linear regulators and it is possible to find more information in works [68], although not always [65], which was undoubtedly influenced by the proper configuration and parameters of the regulators.

In the following years, when the methods of „modern control theory" were developed (controllability and observability analysis, optimal control, adaptive control, robust control with self-learning) and new effective control techniques became popular (H∞/μSynthesis, LQR, SMC, FLC, NN), successful attempts were made to apply them also in the synthesis and analysis of 4WS control systems. Here it is worth mentioning the highlights of the application of these new techniques and examples of their use in the control of 4WS cars:

- The H∞/μ-Synthesis technique is concerned with control that is „robust" to disturbances and parameter uncertainty of the MIMO (multidimensional, i.e., with many inputs and many outputs) linear object under control. This is achieved by appropriately optimising the frequency characteristics (H∞) of standard controllers in a MIMO structure using special software (μSynthesis). Examples of applications give [7, 11, 15]. More information we can find in: [17, 21, 37]. Interesting reflections are provided by [48, 69, 70], too.

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- The LQR (Linar-Quadratic Regulation) technique concerns the optimal regulation of linear (also non-stationary) MIMO objects with an assumed integral quality index based on quadratic forms of the model occurring variables. The LQR technique leads to Kalman regulator algorithms in the MIMO structure. In some simple cases, where the final control time is not imposed, these regulators turn out to be in the form of standard linear regulators (e.g. PID). Examples of applications: [27, 56, 57]. It is possible to get more interesting conclusion in: [58, 59].

- The SMC (Sliding Mode Control) technique refers to the so-called „sliding control" of nonlinear objects by specifying a forcing with a discontinuous form causing the model structure to switch with the control and „slide" the motion trajectory along the desired trajectory. Examples of applications: [23, 40].

- The FLC (Fuzzy Logic Control) technique refers to control using a discrete heuristic model of the behaviour of the controlled object based on so-called fuzzy logic (not Boolean two-state „black-white" type, but multi-state with shades of grey). Examples of applications: [8, 63, 66].

- NN (Neural Network) technique refers to control based on neural networks. Example of applications: [25, 55].

In the synthesis of control algorithms, simple mathematical models of the bicycle model type are usually used, e.g. [26, 60, 68], but sometimes slightly extended, e.g. by adding members expressing the effect of vehicle roll on its movements in the road plane. In the procedure of creating the algorithm of the 4WS controller described in [27, 40, 41], there are also simplified linear models of the driver.

The control tasks described in publications mainly concern the problems of stabilisation of a disturbed (e.g. by wind) motion of a vehicle driving on a straight road, e.g. [54, 59, 67]. Some works discuss 4WS control in driving on a winding road, e.g. [28] and in single or double lane changes, e.g. [30, 31, 40].

The study of 4WS vehicle control systems mainly uses computer simulation and sometimes also HIL (Hardware in the Loop) simulation, where the controller controls through a suitable interface a virtual vehicle whose movement is simulated, e.g. [50, 68]. Standard simulation studies reflecting the operation of 4WS controllers include models of varying complexity [9, 15, 31]. More example it is possible to find in: [35, 38, 48]. It is worth to notice [65], too. An interesting overview of the models used is presented in [19].

The authors rarely reveal the details of the software used. One can think that these are programs using universal programming tools, e.g. Matlab-Simulink and standard numerical procedures. Some works, e.g. [31, 60] mention about the use of ADAMS and DADS software - for MBS simulations or specialized software focused on vehicle motion dynamics studies, e.g. BAMMS [19], CarSim [62]. Very rarely the authors share information where the numerical data for simulation models were used in the calculation procedures.

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The use of modern control techniques coincided with the development of active steering systems, which today can already be found in various automotive applications, including the Lane Keeping System (LKS) to assist the driver in maintaining the lane. Such a system applied to a 4WS vehicle is described on theoretical grounds in the work [52]. The development of this idea towards full automation of the car steering in the 4WS structure requires the controller implementing predictive control (e.g. according to the MPC - Model Predictive Control [17] concept) operating in the two-level structure on the basis of measurements of the waveforms variables describing the movement of the real vehicle. Then the controller, at its higher level, determines on-line from a simple reference model the set signals for the control systems operating at the lower level. The controller is also capable of generating steering control signals, e.g. [6, 7]. A detailed description of the procedure for creating the MPC control algorithm (but for 2WS vehicles) is presented in [62].

In parallel with the development of the methods of steering control of the rear wheels, many researchers have taken up the subject of 4WS control, when the steering control is performed in parallel with the drive/braking control of these wheels. With such extended drift-oriented control (often referred to as DYC - Direct Yaw Control), it is possible to influence the movement of a 4WS vehicle to a much greater extent, also under difficult conditions of wheel interaction with a slippery road. Information on DYC in 4WS structures can be found e.g. in papers [1, 2, 3]. Interesting informations about this topic mention: [20, 36, 42]. It is possible to find more conclusion in: [48, 49, 51], also with optimization elements [46], and the decoupling concept [13]. A further development of the complex control is the linking of 4WS with ACS active suspension systems [32].

After integrating the operation of separate controllers of 4WS, ESP, ABS and ACS in one system and linking the wheel slip control with the prediction of optimal trajectory of the vehicle movement, a new quality of control will be obtained with the perspective of use in autonomous cars.

4. Concept of automatic lane change in 4WS vehicle

The lane change process (of 2WS as well as 4WS vehicles) refers to two variables – the displacement of the centre of mass $Y(t)$ and the angular position of the car body in relation to the trajectory of the centre of mass $\psi(t)$. According to experiences as well as the control theory, the steering system signal $\delta_H(t)$ generated by the driver or automatic controller should have the “bang-bang”-type form and the control process can be divided into two phases – the transposition and the stabilization (Figure 3).

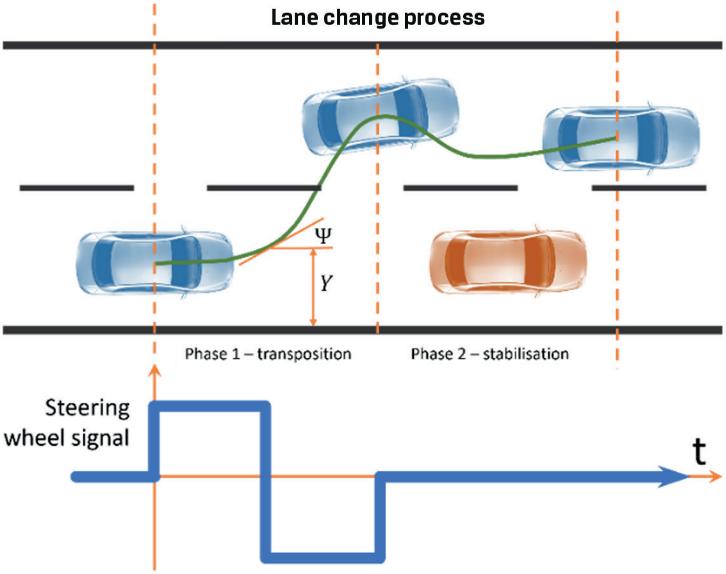
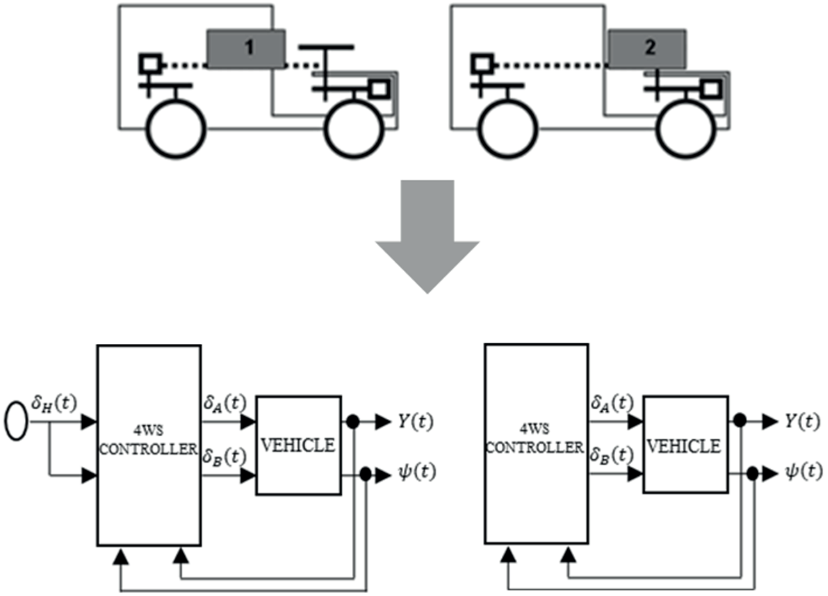


Fig. 3. The concept of time decomposition of lane change control in 2WS vehicle

The two most important general concepts of the 4WS control algorithms are explained by the block diagrams in Figure 4. Block number 1 shows that the controller determines the steering of the rear wheels by measuring the steering angle (given by the driver). Block number 2 shows that the controller determines the steering of the front and rear wheels (autonomous vehicle).

When the steering signal $\delta_H(t)$ is generated by the driver action, the 4WS controller uses this signal as the basis for generating steering signals for the front and rear wheels $(\delta_A(t), \delta_B(t))$. In this system, the decision to initiate the maneuver procedure is made by the driver, so the system can be treated as an assisting system. In the autonomous system, it is done automatically through the road situation monitoring system (not discussed in the study and not marked in the block diagram). In both cases, the controller uses for regulation correction actions an automatic measurement of the lateral displacement $Y(t)$ and angular $\psi(t)$ waveforms.



4WS controller assisting the driver

4WS controller in autonomic car

Fig. 4. Two general structures of 4WS active control system

According to a general concept of automatic control of lane change process the controller contains a reference signals' generator (reference waveforms of control and response signals) and a system of regulators correcting the control signals (responsible for shifting the vehicle to a new track and its angular stabilization). The main idea of the 4WS controller is expressed below (Figure 5) for fully automated cars.

The controller generates two steering system signals $\delta_A(t)$, $\delta_B(t)$ which are input signals for the vehicle's front and rear steering system respectively. This input signals consist of the reference "bang-bang" type signals $\delta_{AR}(t)$, $\delta_{BR}(t)$, modified by corrective signals $\Delta_{\delta A}(t)$, $\Delta_{\delta B}(t)$ from regulators. Of course reference signals $\delta_{AR}(t)$, $\delta_{BR}(t)$ are interdependent (in the simplest form - as in Figure 1). Corrective signals $\Delta_{\delta A}(t)$, $\Delta_{\delta B}(t)$ are done on the basis of error signals between reference and measured signals describing vehicle trajectory $Y_R(t)$-$Y(t)$, $\psi_R(t)$- $\psi(t)$.

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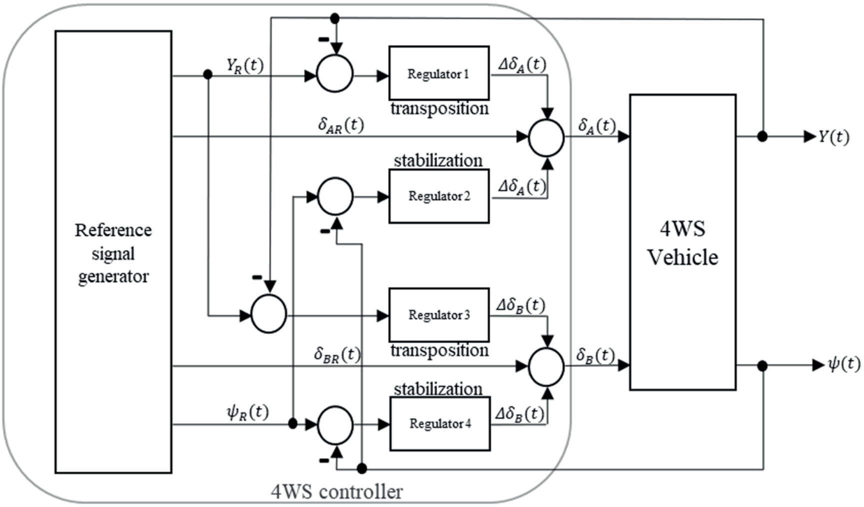

Fig. 5. The concept of lane change control system

A fundamental role in the development and implementation of the 4WS controller is played by the reference model describing the lateral dynamics of the car. Similarly to the above-mentioned works of other authors, the so-called bicycle model of lateral car dynamics (Figure 6) supplemented with equations transforming variables from the local to the global system was adopted as a reference model.

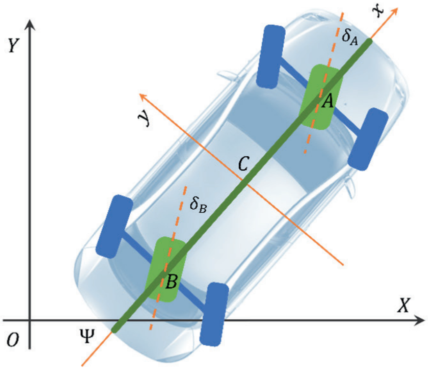

Fig. 6. Idea of the "bicycle model" of 4WS vehicle

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Notation:

- (X, Y), (x,y) – coordinates of the center of mass (p. C) in the local and global coordinate system;
- U – lateral speed in the local coordinate system;
- Ω – yaw angular rate;
- ψ – side slip (yaw) angle;
- δ_A, δ_B – front and rear steering angles;
- δ_H – steering wheel angle;
- V – velocity (here constant);
- m – vehicle mass;
- J – vehicle moment of inertia (Jzz to p.C);
- L_A,L_B - distances AC and BC between the center of mass to the front / rear axis;
- K_A,K_B – front and rear tyre cornering stiffness (yaw coefficients to p. A, B);

Linear equations of motion in the local coordinate system associated with the vehicle (according to the Bolzman-Hamel theory, they apply to the so-called quasi-velocities):

- $$m\dot{U}(t)+\frac{K_A+K_B}{V}U(t)+\frac{mV^2+K_AL_A-K_BL_B}{V}\Omega(t)=K_A\delta_A(t)+K_B\delta_B(t) \qquad (1)$$
- $$J\dot{\Omega}(t)+\frac{K_AL_A{}^2+K_BL_B{}^2}{V}\Omega(t)+\frac{K_AL_A-K_BL_B}{V}U(t)=K_AL_A\delta_A(t)-K_BL_B\delta_B(t) \qquad (2)$$

The equations for the transformation of variables from the local to the global system related to the road:

- $$X(t)=\int_0^t\dot{X}(\tau)d\tau=\int_0^t\big(V\cos(\psi(\tau))-U(\tau)\sin(\psi(\tau))\big)d\tau \qquad (3)$$
- $$Y(t)=\int_0^t\dot{Y}(\tau)d\tau=\int_0^t(V\sin(\psi(\tau))+U(\tau)\cos(\psi(\tau)))d\tau \qquad (4)$$

It is well known from driver practice that steering signals should have a form of short "bang-bang"-type signals for realization of lane change processes. In such case the maximal angle of vehicle deviation is small, so the linearization of the model is possible. In such a case:

$$X(t)=Vt \qquad Y(t)=\int_0^t\big(V\psi(\tau)+U(\tau)\big)d\tau \qquad (5),(6)$$

Then the linearized model of the 4WS vehicle dynamics can be presented as "the black box" form with four Laplace transfer functions expressing actions of two input signals (angles of front and rear vehicle wheels) on two output signals (linear and angular dislocation of the vehicle body) – Figure 7.

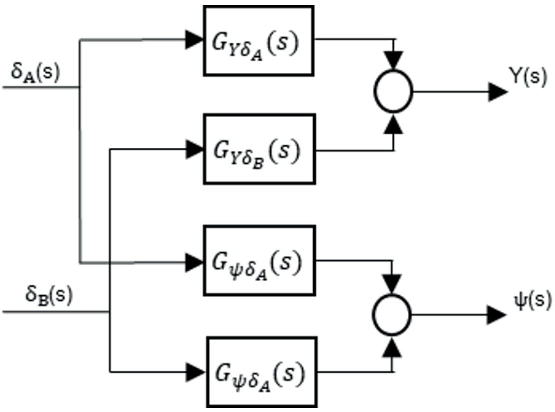

Fig. 7. Idea of transmittance type model

The transmittances appearing in Figure 7 can be represented using standard notation (using transmittance parameters such as gain coefficients, time constants, etc.). An example transmittance $G_{Y\delta_A}(s)$ resulting from the reference model is shown in the following formula:

$$G_{Y\delta_A}(s) = \frac{K_{Y\delta_A}\left(T_{Y\delta_A}^2 s^2 + 2\xi_{Y\delta_A} T_{Y\delta_A} s + 1\right)}{s^2\left(T_0^2 s^2 + 2\xi_0 T_0 s + 1\right)} \tag{7}$$

where

$$K_{Y\delta_A} = \frac{K_A K_B (L_A + L_B) V^2}{K_A K_B (L_A + L_B)^2 - mV^2 (K_A L_A - K_B L_B)} \tag{8}$$

$$T_0 = V\sqrt{\frac{mJ}{K_A K_B (L_A + L_B)^2 - mV^2 (K_A L_A - K_B L_B)}} \tag{9}$$

$$\xi_0 = \frac{m\left(K_A {L_A}^2 + K_B {L_B}^2\right) + J(K_A + K_B)}{2\sqrt{mJ\left(K_A K_B (L_A + L_B)^2 - mV^2 (K_A L_A - K_B L_B)\right)}} \tag{10}$$

$$T_{Y\delta_A} = \sqrt{\frac{J}{K_B (L_A + L_B)}} \qquad \xi_{Y\delta_A} = \frac{L_B}{2V}\sqrt{\frac{K_B (L_A + L_B)}{J}} \qquad (11), (12)$$

Note, that transfer functions parameters are associated with mechanical parameters (vehicle speed, mass, yaw moment of inertia, wheels cornering stiffness, etc.) by analytical formulas. This is important for synthesis of the controller algorithm.

The reference model allows for the determination of the generated reference waveforms $\delta_{AR}(t)$, $\delta_{BR}(t)$ $Y_R(t)$, $\psi_R(t)$ and the determination of controller structures and parameters.

Determination of generated reference waveforms:

The waveforms $\delta_{AR}(t)$, $\delta_{BR}(t)$ are the input signals for the reference model. According to the theory and practice in the process of lane changing, their forms are of the "bang-bang" type, except that their magnitudes are related by a relation (Figure 1). The "bang-bang" course is defined by its parameters - magnitude and duration (time). These parameters can be determined from the transmittance form of the reference model, using the known operator calculus formulas for calculating the values determined from the limits of transformations. The calculations are carried out assuming the preset value of the lateral displacement Y_0 (resulting from the preset lane change) and the preset value of the maximum angular displacement ψ_0 (at the level of several degrees, so that the conditions of linearity of the model are maintained). The waveforms $Y_R(t)$, $\psi_R(t)$ are solutions of the reference model with the given waveforms $\delta_{AR}(t)$, $\delta_{BR}(t)$.

Determination of regulator structures and parameters:

The controller structures and parameters are determined by solving the LQR task defined for the reference model. As it is known, the LQR method requires solving a system of nonlinear algebraic equations of Riccati type resulting from the reference model. Therefore, in order to obtain analytical forms of the solution of the LQR task, the reference model used for the purpose of controller synthesis is reduced by omitting members responsible only for transients, and not for the steady states of the process. As a result, analytical forms of regulator algorithms are obtained. The parameters of these regulators are analytical functions based on the parameters of reduced transmittances, and thus on the mechanical parameters of the reference model.

Both the determination of the generated reference waveforms and the determination of the structures and parameters of the controllers require the knowledge of the mechanical parameters of the reference model and the assumed parameters of the "bang-bang" control of the lane change. Therefore, it is necessary to carry out extensive simulation studies using a virtual model of the vehicle, whose structure and parameters would slightly differ from the structure and parameters of the reference model assumed in the synthesis of the controller algorithm.

5. Conclusion

The study of 4WS control carried out on the basis of literature materials allows us to state that the problem is very extensive, multi-threaded and scientifically very attractive. In spite of many research works carried out in various centres in the last forty years, which resulted in numerous publications, it is safe to say that a number of interesting issues directly related to the 4WS control have been poorly explored or even not addressed yet.

First of all, such an issue seems to be the active control of a 4WS vehicle during sudden manoeuvres performed at considerable speeds, i.e. when the dynamics of traffic is of prime importance. This concerns, for example, an abrupt change of lane when an obstacle suddenly appears in front of a fast-driving car, braking cannot avoid a collision, and the only salvage is an evasive manoeuvre. It is worth considering various control algorithms, bearing in mind, on the one hand, various concepts of the 4WS controller, sensor system and mechanical components, and on the other hand, references to various variants of car dynamics properties resulting from its parameters and characteristics.

The statements made here quite clearly show the directions of further work related to the control of 4WS. These should include:

- determination of 4WS control algorithms using a reference bicycle model and variables describing the motion of a real vehicle;
- development of reference model parameter identification methods;
- development of a virtual model of the 4WS controlled by the prepared algorithms, the model taking into account the imperfections of measurement and signal processing and the imperfections of mechanism operation;
- development of simulation software allowing for testing of control algorithms and conducting analysis of their sensitivity (due to errors in estimating parameters of the reference model, due to imperfections of the measurement signals and their processing etc.);
- conducting extensive simulation studies and sensitivity analysis of the developed control algorithms.

The 4WS control algorithms developed in this way should be an attractive proposal for designers of real controllers. The resulting studies will significantly enrich the knowledge of car dynamics and control.

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